The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 33

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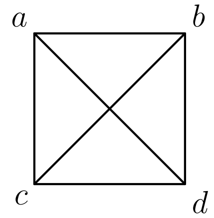
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 169–172 (1792).]

PROPOSITION XXXIII. Theorem XXIII.

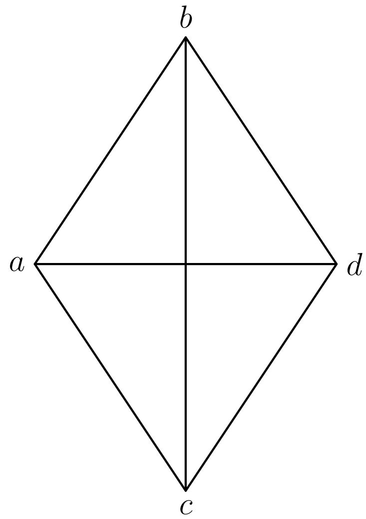
The right lines which join equal and parallel right lines at the same parts, are themselves also equal and parallel.

The present theorem is, as it were, the confine of the consideration of parallels and parallelograms: for it seems to declare a certain symptom of parallel right lines, and delivers the latent origin of parallelograms. For a parallelogram is formed, as well from those equal and parallel right lines, which are drawn in the beginning, as from those which conjoin them, and which are in like manner shewn to be equal and parallel. Hence, the proposition which immediately follows the present, contemplates the properties essentially inherent in these spaces, in a parallelogram as it were already constructed. And these things are indeed manifest. But it is requisite to consider the diligence which this proposition contains. In the first place, indeed, that it is not sufficient, that the lines which are conjoined should be equal: for the lines which connect equals, are not entirely equal, unless they are also parallel. For a triangle being isosceles, and a point being assumed in one of the equal sides, and through this a line being drawn parallel to the basis, equal lines shall indeed conjoin parallels to the basis, and the basis itself, yet thes parallels shall not also be equal; and the sides will not be parallel, because they coincide at the vertex of the triangle.

In the second case, he considers that the subject right lines being parallel, is not sufficient to constitute the equality of the lines which conjoin them. For this is evident from the preceding construction of the isosceles triangle; since the drawn right line, and the basis, are parallel, and yet the lines which connect them are not parallel, because they are parts of the sides of the isosceles triangle. The parallel position, therefore, of the lines which are conjoined, is requisite to the equality of the connecting lines: but the equality of the latter is necessary to the parallel position of the former. On this account the institutor of the Elements assumes each, in those which are conjoined, for the purpose of exhibiting, that the connecting lines are as well equal, as parallel to one another. But in the third place, he intimates, that right lines being supposed both equal and parallel, their connecting lines will not be universally equal and parallel. For unless we make the conjunctions at the same parts as in this case, the connecting lines cannot be parallel (since they will cut each other), so they may be sometimes equal, and sometimes not. For if you assume a quadrangle, or oblong, as $a\,b\,c\,d$, and connect the right lines $a\,d$, $b\,c$, the diameters are indeed equal, but not parallel, and they conjoin the equal and parallel opposite sides of the aforesaid spaces. But



if the figure be a Rhombus, or a Rhomboides, the diameters of these, are not only non-parallels, but also unequal. For since $a\,b$, is equal to $c\,d$, but $a\,c$ is common, and the angle $b\,a\,c$, is unequal to the angle $a\,c\,d$, the bases also are unequal The institutor of the Elements, therefore, very properly



considered, that the lines which conjoin equal and parallel lines, ought to make the conjunction at the same parts, lest $a\,c$, $b\,d$, being supposed equal and parallel, we should assume $a\,d$, $b\,c$, as the connecting lines, and not $a\,b$, and $c\,d$. For he shews that these latter are equal and parallel: but that the former are, indeed, never parallel, but equal, as we have observed in a quadrangle and oblong, but never in a rhombus and rhomboides; as the opposite to this has been proved to be true, because they are unequal, on account of the inequality of the angles internal, and situated at the same parts.