The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 30

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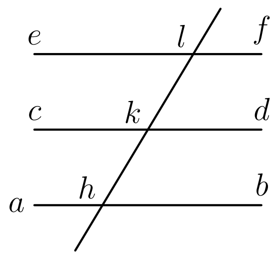
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 159–161 (1792).]

PROPOSITION XXX. Theorem XXI.

Right lines parallel to the same right line, are parallel to each other.

The geometrician in these discourses which are conversant with relation, is accustomed to shew identity permeating through all quantities, having the same relation to the *same*. Thus among the axioms also he says, *things equal to the same, are equal to each other*: and in the following books he says, *things similar to the same, are similar to each other*, and *things having the same proportion to the same, have the same proportion to each other*. After this manner, therefore, he now also demonstrates, *that right lines parallel to the same, are parallel to each other*. But it happens that this is not true in all respects. For quantities double of the same, are not also double of each other: nor are those which are sesquialter of the same, sesquialter likewise to each other, but it appears to take place in those alone, which are univocally converted in equality, similitude, identity and parallel position. For that which is parallel to a parallel, is itself also parallel. As that which is equal to an equal, is itself equal; and that which is similar to a similar, is itself similar. For the relation of parallels to each other, is similitude of position. He affirms, therefore, and shews, in the present theorem, that lines parallel to the same, are entirely so related, that they are also parallel to each other. And he also exhibits the parallels with an external position, and likewise a medium, to which these have a similar relation, that what he asserts may become manifest from a common conception. For if they coincide with each other on either side, and coincide with that which is situated in the middle, they will no longer be parallel to it.

But it is possible that he who changes the position may shew the same thing, and by the same methods which the geometrician employs in exhibiting his proposition. For instance, he assumes both $c\,d$, and $e\,f$, parallel to $a\,b$, both of them situated above, and $a\,b$ being beneath, and not in the middle. For a right line $h\,k\,l$, falling upon them, makes each of the angles $h\,k\,d$, $k\,l\,f$, equal to $a\,h\,k$, because they are alternate; and on this account it makes the angles $h\,k\,d$, $k\,l\,f$, equal to each other. The right lines, therefore, $c\,d$, $e\,f$, are parallel. But if any one should say that $a\,h$, $h\,b$, are parallel to $c\,d$, and are therefore parallel to each other, we reply that $a\,h$, $h\,b$, are parts of one parallel, and are not two parallel lines. For parallels are conceived to be



infinitely produced, but $a\,h$, when produced, falls upon $h\,b$. It is therefore the same with $h\,b$, and not a different line. Hence, all the parts of a parallel, are parallel both to the right line, to which the whole was parallel, and to its parts. As for example, $a\,h$ is as well parallel to $k\,d$, as $h\,b$, to $c\,k$. For if they are infinitely produced, they will never coincide. And these remarks must be considered as not foreign from the purpose, both on account of sophistical importunities, and the juvenile habits of mathematical auditors. For the vulgar rejoice to find captious reasonings of this kind, and to procure vain molestation to the possessors of science. Biut it is not requisite to convert the present theorem, and to shew that lines parallel to each other, are also parallel to the same. For if we again suppose one line parallel to to some other, the remainder also of these shall be parallel to it, and they will be parallel to the same, and we shall return to the same proposition.