# The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 28

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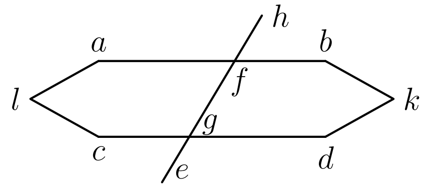
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 150–152 (1792).]

PROPOSITION XXVIII. THEOREM XIX.

> If a right line falling upon two right lines, makes the external equal to the internal angle, placed opposite, and at the same parts, or makes the angles internally situated, and at the same parts equal to two right, those right lines shall be parallel to each other.

The preceding theorem receiving the angles, not at the same parts, but situated between right lines, shews that the right lines are parallel among themselves: but the present theorem proposes the two remaining hypotheses, of which one separates the angles according to the particles *without* and *within*, but the other supposes them both within, and exhibits the same conclusion. But it may seem, perhaps, that the institutor of the Elements has inconveniently distributed the theorems. For it was necessary either to receive three hypotheses in a divided manner, and to make three theorems; or to collect all into one theorem, as Æneas Hierapolites does, who wrote a compendium of the Elements; or willing to divide them into two, to make an orderly division, and to assume the hypotheses separately, which contain equal angles, and separately that in which the angles are equal to two right. But in the present propositions, in one theorem he supposes the alternate angles equal, but in the other, the external to the internal, and the internal angles situated at the same parts equal to two right. What then is the cause of this division? Does he regard the equality of the angles to each other, or to two right, and on this account does not separate the proposed theorems from each other; or does he respect the angles being received at the same, or not at the same parts? For the preceding theorem does not respect angles at the same parts, since such as these are alternate: but the present regards such as are situated at the same parts, as is perspicuous from the proposition. But how the institutor of the Elements shews, that from the internal angles being equal to two right, the right lines are parallel, appears from his writings on this subject. Ptolemy, however, inthe theorems in which he proposes to demonstrate that right lines produced from angles less than two right, coincide at the same parts, in which the angles less than two right are situated, shewing before all his theorems, that from the internal angles being equal to two right, the right lines are parallel, proves it in the following manner. Let there be two right lines $a\,b$, $c\,d$, and let a certain right line $e\,g\,f\,h$, so cut them, that it may make the angles $b\,f\,g$, and $f\,g\,d$, equal to two right, I say that those right lines are parallel, that is, will never coincide. For if it be possible, let them coincide while the right lines $b\,f$, $g\,d$,




are produced in the point $k$. Because, therefore, the right line $e\,f$, stands upon the right line $a\,b$, it makes the angles $a\,f\,e$, $b\,f\,e$, equal to two right. In like manner because $f\,g$ stands on $c\,d$, it makes the angles $c\,g\,f$, $d\,g\,f$, equal to two right. Hence, the four angles $b\,f\,e$, $a\,f\,e$, $c\,g\,f$, $d\,g\,f$, are equal to four right, two of which $b\,f\,g$, $f\,g\,d$, are supposed equal to two right. The remainders, therefore $a\,f\,g$, $c\,g\,f$, are equal to two right. If then the right lines $f\,b$, $g\,d$, when produced, coincide, the internal angles being equal to two right, $f\,a$, and $g\,c$, also, shall coincide when produced: for the angles $a\,f\,g$, $c\,g\,f$, are also equal to two right. Either therefore the right lines shall coincide in both parts, or in neither, since these, as well as the former, are equal to two right. Let the right lines then $f\,a$, $g\,c$, coincide in the point $l$. But if this be admitted two right lines $l\,a\,f\,k$, $l\,c\,g\,k$, will comprehend space, which is impossible. It is not therefore possible, that the internal angles being equal to two right, the right lines can coincide. They are therefore parallel.