The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 27

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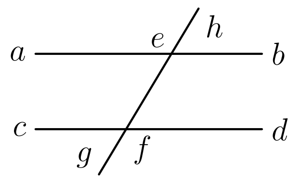
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 146–150 (1792).]

PROPOSITION XXVII. Theorem XVIII.

If a right line falling upon two right lines, makes the alternate angles equal to each other, those right lines shall be parallel to each other.

In the present theorem it was not pre-assumed as evident that the right lines are in one plane, but this ought rather to be previously admitted in all theorems which are considered in a plane. This, however, is added, because it does not universally follow, that when the alternate angles are equal, the right lines will be parallel, unless they are in the same plane. For nothing hinders, but that a right line falling on right lines disposed in the shape of the letter X, one of which is situated in one plane, but the other in a different one, make make the alternate angles equal; and yet the right lines thus disposed will not be parallel. It was pre-assumed[170], therefore, that in a treatise on planes, we conceive every thing described in one and the same plane: and on this account, he does not require this addition in the present proposition. But it is requisite to know that the geometrician considers the particle *alternate*, in a twofold respect, sometimes, indeed, according to a certain situation, but sometimes according to a certain consequence of proportions. And according to this last signification, the particle *alternate* is used in the fifth book, and in such as are arithmetical: but agreeable to the former, both in this, and in all the other books concerning parallel right lines, and that which falls upon these. For he calls the angles alternate, which are not formed at the same parts, and are not successive to each other, which are distinct, indeed, from the incident line, but both of them exist within parallels, and differ in this, that the one has an upward, but the other a downward position. I say, for example, that when a right line $e\,f$, falls on the right lines $a\,b$, and $c\,d$, he calls the angles $a\,e\,f$, $d\,f\,e$, and also the angles $c\,f\,e$, $b\,e\,f$ *alternate*, or *altern*, because they have an alternate, or changed order, according to their position. But this too must be known, that from such a situation of right lines, all the symptoms become by division, six; three of which the geometrician alone receives; and three he omits. For we either assume the angles at the same parts, or not at the same. And if at the same parts, either both within the right lines, which shews them to be parallels; or both

[170]In Book II. Comment 2. of this work.



without, or one without, and the other within. And if not at the same parts, again, after the same manner, they are either both without the right lines, cutting the lines it is necessary to receive; or within; or one within, and the other without. But what we have said will become manifest by the same description as above. For let there be certain right lines $a\,b$, $c\,d$, and let a right line $e\,f$ fall upon them, and let it be produced to the points h and g. If then you assume angles at the same parts, you will either place them both within, as $b\,e\,f$, and $e\,f\,d$, or as $a\,e\,f$, and $e\,f\,c$; or both without, as $h\,e\,b$, and $d\,f\,g$, or as $h\,e\,a$, and $c\,f\,g$; or one within, and the other without, as $h\,e\,b$, and $e\,f\,d$, or as $g\,f\,d$, and $f\,e\,b$, or as $h\,e\,a$, and $e\,f\,c$, or as $g\,f\,c$, and $a\,e\,f$: for these last are received in a quadruple respect. But if you assume the angles not at the same parts, you will either place both within, as $a\,e\,f$, and $e\,f\,d$, or as $c\,f\,e$, $f\,e\,b$; or both without, as $a\,e\,h$, and $d\,f\,g$, or as $h\,e\,b$, and $c\,f\,g$; or one within, and the other without, and this again in a quadruple respect. For they will either be the angles $a\,e\,h$, and $e\,f\,d$; or $h\,e\,b$, and $e\,f\,c$; or $g\,f\,c$ and $f\,e\,b$; or $g\,f\,d$, and $f\,e\,a$. And besides these, there is no other asumption.

As, therefore, angles are assumed according to six modes, the geometrician combines three assumptions alone; and these consequent symptoms, are naturally adapted to express parallels. But of these three assuptions, one belongs to those angles which are not at the same parts, viz. to those which are only assumed within; and these he calls alternate, so that those, which are both external, and those, one of which is external, but the other internal, are omitted: but two of these assumptions belong to angles at the same parts, to those, indeed, which are both internal, which he says are equal to two right, and to those, one of which is internal, but the other external, which he says are equal, leaving indeed one assumption which supposes both the angles to be external. We therefore affirm that the same things will be consequent to the three omitted hypotheses. Thus, in the preceding figure, let both the external angles $h\,e\,b$, $d\,f\,g$, be at the same parts, I say that these are equal to two right angles. For the angle $d\,f\,e$, is equal to the angle $h\,e\,b$, and the angle $b\,e\,f$, to the angle $d\,f\,g$. But if the angles $b\,e\,f$, $e\,f\,d$, are equal to two right, the angles $d\,f\,g$, $h\,e\,b$, are equal to two right. Let again the angles $a\,e\,h$, $e\,f\,d$, not be towards the same parts, of which the one is within, but the other external, I say that these also are equal to two right angles. For if the angle $a\,e\,h$, is equal to the angle $b\,e\,f$, but the angles $b\,e\,f$, and

$e\,f\,d$, are equal to two right, the angles, also, $a\,e\,h$, and $e\,f\,d$, are equal to two right. Again, let them not be at the same parts, but both without the right lines as $a\,e\,h$, $d\,f\,g$. I say that these are equal to one another. For if the angles $a\,e\,h$, and $b\,e\,f$, are equal to each other, but the angle $d\,f\,g$, is equal to the angle $b\,e\,f$, hence the angle $a\,e\,h$, is not unequal to the angle $d\,f\,g$. If, therefore, the things assumed by the geometrician, in three hypotheses are verified, all the same follow in the remaining three as indisputably true. Besides this too is to be observed, that in such as the geometrician receives these, according to two assumptions, the angles are supposed equal to each other, but when according to one assumption, equal to two right: but in these last on the contrary, according to two assumptions, they are supposed equal to two right angles, but according to one equal to each other. For since all the assumptions are six, it happens, indeed, from three, that the angles are equal to two right, but from the other three, that they are equal to each other. Hence, those which are omitted are not undeservedly contrary to the assumptions which are reckoned worthy of relation. But the geometrician appears to have chosen such hypotheses as either abound in affirmation, or are more simple, and on this account of those angles which are not at the same parts, he assumed alone the internal, which he calls alternate: but of those at the same parts, he assumes as well the internal, as well as one internal and the other external: but he avoids the rest, either because they are more declared by negation, or because they are more various. However, whether this or some other be the cause, the number of consequents to those hypotheses is from hence sufficiently manifest.