The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 26

Transcribed by David R. Wilkins

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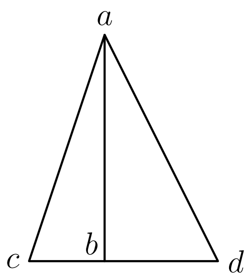
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 139–139 (1792).]

PROPOSITION XXVI.[167] Theorem XVII.

If two triangles have two angles equal to two, each to each, and one side equal to one side, either that which is adjacent to the equal angles, or that which subtends one of the equal angles: then they shall have the remaining sides equal to the remaining sides equal to the remaining sides, each to each, and the remaining angle equal to the remaining angle.

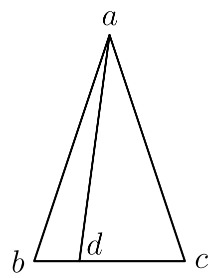
It is necessary, that he who wishes to compare triangles with each other, according to sides, angles and areas, should either, by receiving the sides alone equal, enquire after the equality of angles; or by assuming the angles alone equal, investigate the equality of the sides; or by mingling the angles and sides, scrutinize the equality of angles and sides. Since, therefore, Euclid alone receives the angles equal, he could not likewise shew that the sides of the triangles are equal. For the least triangles are equiangular with the greatest, though at the same time they are excelled by them, both according to sides and comprehended space: but the angles of the former are separately equal to the angles of the latter. However, as he supposes the sides alone to be equal, he demonstrates that all are equal, by the eighth theorem, in which there are two triangles having two sides equal to two, each to each, and the base to the base, and these are shewn to be equiangular, and to possess a power of comprehending equal spaces. And the institutor of the Elements omits this addition, as necessarily following from the fourth, and requiring no demonstration. But when receiving sides and angles, he ought to receive either one side equal to one; or one side, and two angles of the triangles, equal to two; or on the contrary, one angle and two sides; or one angle and three sides; or one side and three angles; or more than one side, and more than one angle. But when he had received one angle and one side, he could by no means shew the thing proposed. I mean, the equality of the rest. For it is possible that two triangles which are equal, according to one side only, and one angle, may be entirely unequal as to the rest. Thus let there be a right line $a\,b$, perpendicularly erected upon the right line $c\,d$, but let $b\,d$ be greater than $b\,c$, and connect $a\,c$, $a\,d$. In these triangles therefore,

[167] [DRW—printed 'PROPOSITION XV' in 1792 text.]

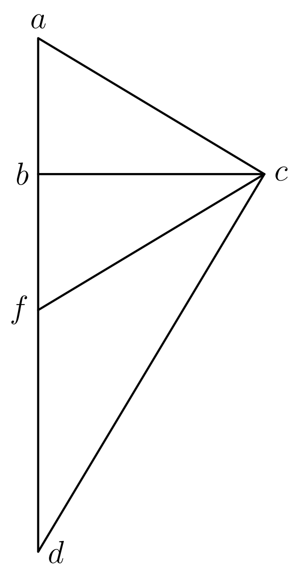


there is one common side, and one angle equal to one, but all the rest are unequal. But it is lawful to receive one side, and two angles, and to prove the rest equal, and this he performs by the present theorem: though again, to suppose one side, and three equal angles, is superfluous; since from the equality of two alone, the equality of the rest is exhibited. Agan, receiving one angle, and two equal sides, he demonstrates that the rest are equal in the fourth theorem. But it is superfluous to receive one angle, and three equal sides: for two equals being alone assumed, conclude the equality of the rest. Besides, it is superfluous to assume two sides, and two equal angles; or two sides, and three equal angles, or two angles and three sides; or three angles and three sides. For the consequents to fewer hypotheses attend likewise a greater multitude, while the hypotheses are received with proper conditions. Hence, three hypotheses requiring demonstration, present themselves to our view, one, which alone receives three sides; and another which assumes one side, and two angles, which the geometrician now proposes; and a third, the opposite to this. On this account, we have only these three theorems, concerning the equality of triangles, which are conversant in sides and angles; since all the other hypotheses are either invalid for the purpose of shewing the object of enquiry; or they are valid indeed, but superfluous, because the same things may be readily procured by fewer hypotheses. As, therefore, when he assumed two sides equal to two, and one angle equal to one, he did not, indeed, assume every angle, but (as it was proposed by him) that contained by equal right lines, in the same manner when he assumes two angles equal to two, and one side to one, he does not assume any side, but either that which adjacent to the equal angles, of that which subtends one of the equal angles. For neither is is possible in the fourth theorem, by assuming any equal angle, nor in the present by assuming any side, to shew the equality of the rest.

Thus for example, an equilateral triangle $a\,b\,c$, being given, let the side $b\,c$ be divided into unequal parts, by the line $a\,d$. Hence, there will be formed two triangles, having two sides $a\,b$, $a\,d$ equal to the two $a\,c$, $a\,d$, and one angle at the point b, equal to one angle at the point c, but the remaining sides will not



also be equal, as for instance, the side $b\,d$ to the side $d\,c$: for they are unequal. But neither are the remaining angles equal: the reason of which is, because we receive an angle equal to an angle, but not the angle which is contained by equal sides. After the same manner, indeed, the present theorem also will appear dubious, unless we assume, according to the aforesaid condition, an equal side subtending one of the equal angles, or adjacent to the equal angles. For let there be a right angled triangle $a\,b\,c$, having the angle at the point b right, and the side $b\,c$, greater than the side $b\,a$, and let there be constructed on the right line $b\,c$, and at a point in it c, an angle $b\,c\,d$, equal to the angle $b\,a\,c$, and let $b\,d$, $c\,d$, produced, coincide in the point d. There are two triangles, therefore $a\,b\,c$, $b\,c\,d$, having one side $b\,c$ common, and



two angles equal to two, viz. $a\,b\,c$, to $c\,b\,d$ (for they are right), and $b\,a\,c$ to $b\,c\,d$, according to construction. Hence, as it appears the triangles are equal, and yet it may be shewn that the triangle $b\,d\,c$, is greater than the triangle $a\,b\,c$. But the reason of this is, because in the triangle $a\,b\,c$, we assume the common side $b\,c$, subtending one of the equal angles, viz. the angle at the point a: but in the triangle $b\,c\,d$, we assume the equal side, adjacent to the equal angles. It was requisite, therefore, in each, either to subtend one of the equal angles, or to be adjacent to the equal angles. But not observing this, we affirmed that triangle to be equal, which is necessarily greater: for is not the triangle $b\,c\,d$, greater than the triangle $a\,b\,c$? To be convinced of this, let there be constructed on the right line $b\,c$, and at a given point in it c, an angle $f\,c\,b$, equal to the angle $a\,c\,b$: for the angle $b\,c\,d$, as well as the angle at the point a, is greater than the angle $a\,c\,b$. Because, therefore, there are two triangles $a\,b\,c$, $b\,c\,f$, having two angles $a\,b\,c$, $b\,c\,a$, equal to two $c\,b\,f$, $b\,c\,f$, each to each, and one side common, adjacent to the equal angles, viz. $b\,c$, the triangles are equal. But the triangle $b\,c\,d$ is greater than the triangle $b\,c\,f$, and consequently it is also greater than the triangle $a\,b\,c$. But it was formerly shewn to be equal, on account of the assumption of any side: And thus much the diligence of Porphyry has supplied us on the present occasion. But Eudemus, in his Geometrical Narrations, refers the present theorem to Thales. For he says it is necessary to use this theorem in determining the distance of ships at sea, according to the method employed by Thales in this investigation. But from the preceding division we may briefly assume all the contemplation concerning the equality of triangles, and are enabled to relate the causes of things omitted, confuting those hypotheses, as either false, or superfluous. And thus far we determine the limits of the first section of the elementary institutor, because he forms the constructions and comparisons of triangles, according to equal and unequal. And by construction, indeed, he delivers their essence: by the comparison, their identity and diversity. For there are three things which are conversant about being, *essence*, *same*, and *different*[168], as well in quantities, as in qualities, according to the propriety of subjects. From these, therefore, as images it may be shewn, that every thing is the *same* with itself, and *differs* from itself, on account of the multitude which it contains; and that all things are the *same* with one another, and *different* from themselves. For both, in every triangle, and in more triangles than one, equality and inequality has been found to reside.

[168]See more concerning these universal genera in the third section of the Dissertaion, Vol I. of this work.