The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 25

Transcribed by David R. Wilkins

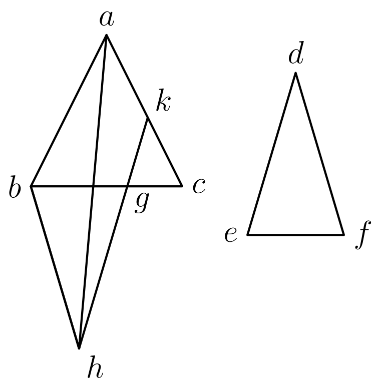
August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 136–139 (1792).]

PROPOSITION XXV. Theorem XVI.

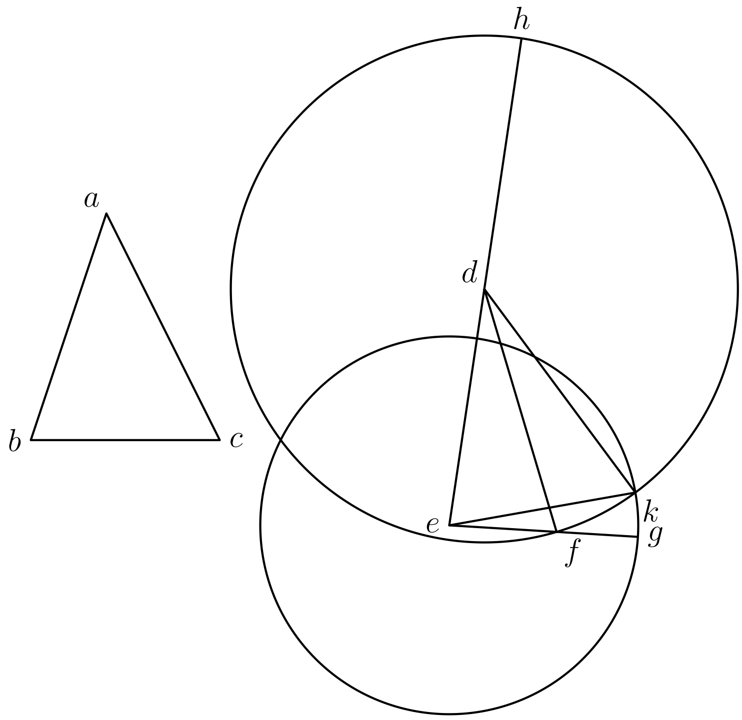
If two triangles have two sides equal to two, each to each, but have the base of the one greater than the base of the other; they shall likewise have the angle contained by the equal sides in one, greater than the angle contained by the equal sides in the other.

The present theorem is the opposite to the eighth, but the converse of the preceding. For the institutor of the Elements produces theorems concerning the equality and inequality of angles and bases, according to *conjunction*; in each of the *conjunctions*, receiving some as precedents, but others as converse. And in such as are precedent indeed, he employs direct ostensions: but in such as are converse, he uses deductions to an impossibility. After this manner he proceeds in some particular triangle, sometimes from the equality of the sides which it contains, shewing the consequent equality of the angles which they subtend: but sometimes from their inequality evincing inequality. And again, on the contrary, affirming that equality of sides is consequent to equality of angles, but inequality to inequality. However, that we may proceed to the thing proposed, we refer those who are desirous of learning how the geometrician shews when this is manifest, to his books on this subject. But we shall briefly relate the demonstrations which others produce of this proposition; and in the first place, that which Menelaus Alexandrinus invented and delivered. Let there be two triangles $a\,b\,c$, $d\,e\,f$, having the two sides $a\,b$, $a\,c$, equal to the two $d\,e$, $d\,f$, each to each, and the base $b\,c$, greater than the base $e\,f$, I say that the angle at the point a, is greater than the angle at the point d. For let there be cut from the base $b\,c$, a line $b\,g$, equal to the base $e\,f$, and construct at the point b, an angle $g\,b\,h$, equal to the angle $d\,e\,f$, and place $b\,h$ equal to $d\,e$. Lastly, connect $h\,g$, and produce it to the point k, and connect $a\,h$. Because, therefore $b\,g$ is equal to $e\,f$, but $b\,h$ to $e\,d$, the two are equal to the two, and they contain equal angles. Hence, $g\,h$ is equal to $d\,f$, and the angle $b\,h\,g$, is not unequal to the angle $e\,d\,f$. And because $g\,h$ is equal to $d\,f$, but $d\,f$ to $a\,c$, $g\,h$, also, is equal to $a\,c$. Hence $h\,k$ is greater than $a\,c$, and consequently is much greater than $a\,k$. The angle, therefore, $k\,a\,h$, is greater than the angle $k\,h\,a$. Again, because $b\,h$, is equal to $a\,b$, for it is equal to $d\,e$, the angle $b\,h\,a$, is equal to the angle $b\,a\,h$. Hence,



the whole angle $b\,h\,k$, is less than the whole, $b\,a\,c$, but it is shewn to be equal to the angle at the point d. The angle, therefore, $b\,a\,c$, is greater than the angle at the point d. And such is the demonstration of Menelaus.

But Heron, the mechanist, shews the same thing, in the following manner, without leading to an impossibility, as is the case with the demonstration of Euclid. Let there be two triangles $a\,b\,c$, $d\,e\,f$, with the same hypotheses as above. And because $b\,c$ is greater than $e\,f$, let $e\,f$ be produced, and place $e\,g$ equal to $b\,c$; and in like manner extend $d\,e$, and place $d\,h$ equal to $d\,f$. The circle, therefore, which is described with centre d, and interval $d\,f$, will pass also through the point h. Let it be described as $f\,k\,h$. And because $a\,c$, $a\,b$,



are together greater than $b\,c$, but these are equal to $e\,h$, and $b\,c$ is equal to $g\,e$, hence the circle which is described with the centre e, but interval $e\,g$, will cut $e\,h$. Let it cut $e\,h$, as the circle $g\,k$, and connect from the common section of the circles to the centres, the right lines $k\,d$, $k\,e$. Because, therefore, the point d, is the centre of the circle $h\,k\,f$, $d\,k$ is equal to $d\,h$, i.e. to $a\,c$. Again, because the point e, is the centre of the circle $g\,k$, the line $e\,k$, is equal to $e\,g$, i.e. to $b\,c$. Hence, since the two $a\,b$, $a\,c$, are equal to the two $d\,e$, $d\,k$, and the base $b\,c$, is equal to the base $e\,k$, the angle, also $b\,a\,c$, is equal to the angle $e\,d\,k$. And thus the angle $b\,a\,c$, is greater than the angle $f\,d\,e$.