The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 23

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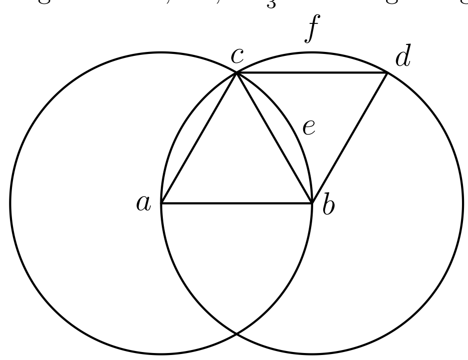
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 126–129 (1792).]

PROPOSITION XXIII. Problem IX.

On a given right line, and at a given point in it, to constitute an angle equal to a given right lined angle.

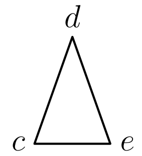
This also is a problem, whose invention according to Eudemus is rather the gain of Oenopides than of Euclid: but it requires the construction of an angle, on a given right line; and at a given point in it equal to another right lined angle. This, then, Euclid necessarily adds, that the given angle must be rectilineal; because it is impossible that an angle can be constructed on a right line equal to every angle. For has been shewn[162] that there are only two curve-lined angles equal to right-lined angles, viz. the angle of a lunular figure, which we have proved equal to every right-lined angle; and the angle of that figure similar to an axe[163], which is equal to two thirds of a right angle. But a lunular figure of this kind, which is called (πελεκοειδες) Pelecoides, is formed from two circles cutting each other through their centres. However, the construction of an angle on a certain right line, causes the constituted angle to become determinate, and not indifferent in species, but forms it either right-lined, or mixt. But since no mixt can be equal to a right-lined angle, it is manifest that this must be perfectly rectilineal. The institutor of

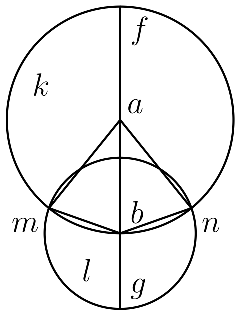
[162]This will be manifest from the following figure. Let the circles $a\,c$, $b\,d$, be drawn passing through their respective centres a, b; and from the centre c, with the radius $c\,b$, equal to $a\,b$, describe the arch $a\,b\,d$, and draw the lines $c\,b$, $c\,d$, $c\,a$. Then because $a\,c\,b$ is an equilateral triangle, as also $c\,b\,d$, each of the angles $a\,c\,b$, $b\,c\,d$, shall be equal to $\frac{2}{3}$ of one right angle; and because the biline $c\,d$, is equal to the biline $c\,b$, hence, the angle formed by the arch $c\,b$, and the arch $c\,d$, viz. the angle $e\,c\,f$, shall be equal to the angle formed by the right line $c\,b$, and the right line $c\,d$, i.e., to $\frac{2}{3}$ of one right angle. Q.E.D.



[163]In the second comment of this book.

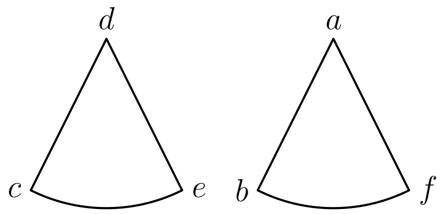
the Elements, therefore, simply using the present problem, and constructing a triangle from three right lines, equal to three given lines, accomplishes the thing proposed. But you may receive a more exquisite construction of the triangle, by the following method. Let there be a given right line ab, and a given point in it a, and a given right lined angle cde. It is required, therefore, to accomplish the problem. Connect ce, and produce ab on both sides to the points f, g. Then place fa, equal to cd, and de to ab, and bg to ec. And with centre a, but interval af, describe the circle k. And again, as in the preceding, with the centre b, but interval bg, describe the circle l. The circles, therefore, will cut each other, as we have shewn in the





last proposition. Let them cut each other in the points m, n, and from these points draw right lines to the centres as in the figure. Because, therefore fa, is equal to am, and an, but cd, is equal to fa; am, and an, will be each equal to cd. Again, because bg is equal to bm, and bn, but gb is not unequal to ce; bm, and bn, will be also equal to ce. But ab is equal to de. The two therefore, ab, am, are not unequal to the two de, dc, and the base bm, is equal to the base ce. Hence, the angle mab, is equal to the angle at the point d. And again, the two na, ab, are equal to the two cd, de, and the base nb, is equal to the base ce. The angle, therefore, nab, is equal to the angle cde, and the thing proposed is doubly accomplished: for we have not only constituted one, but two angles, equal to the given angle, on each side of the right line ab; so that in whatever part we may desire the construction to be made, it will be indubitable, and without contradiction. And this we have added to the construction of the elementary institutor.

But we cannot praise the method of Apollonius, because it requires the assistance of the third book. For he receives any angle cde, and a right line $a\,b$, and with the centre d, but interval $c\,d$, he describes a circumference $c\,e$. In like manner with the centre a, but interval $a\,b$, he describes a circumference $b\,f$; and intercepting a circumference $c\,e$, equal to $b\,f$, he connects the right line $a\,f$, and affirms that the angles a and d, insisting on equal circumferences, are equal. But it is necessary to pre-assume that $a\,b$ is



equal to $c\,d$, in order that the circles may be also equal. We therefore think that a demonstration of this kind requiring posterior propositions, is foreign from an elementary institution; and we give the preference to that of the geometrician, as consequent to principles.