The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 22

Transcribed by David R. Wilkins

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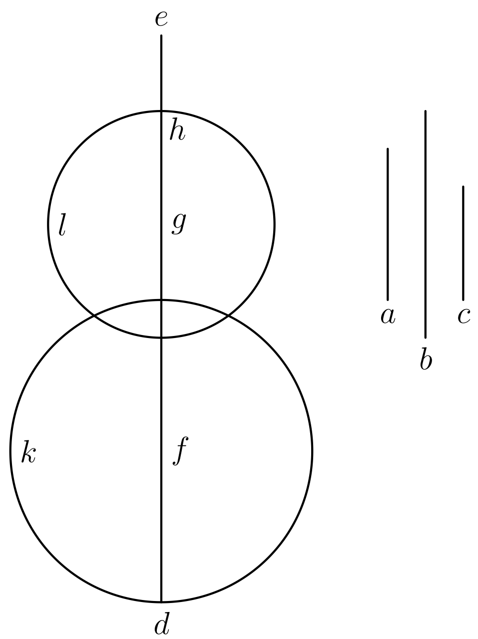
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 122–125 (1792).]

PROPOSITION XXII. Problem VIII.

> To construct a triangle from three right lines, which are equal to three given right lines. But it is requisite that two of the lines must be greater than the remaining one, in whatever manner they may be taken.

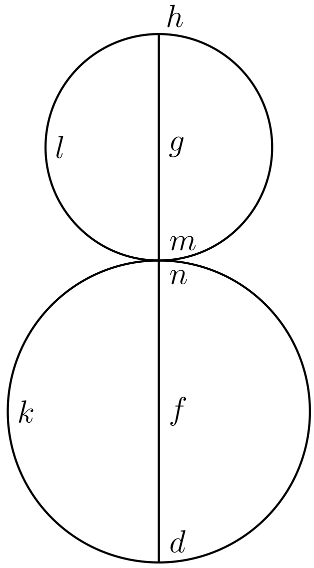
We again pass to problems, and Euclid commands us to construct a triangle from three proposed right lines, two of which are greater than the remaining one, equal to given right lines. Because he knew this in the first place, that it was impossible to construct a triangle from those same lines, which had already received the declared position: but that this was possible to be effected from their equals. In the next place, he knew it was necessary that two of the right lines about to complete the triangle, should be greater than the remaining one: for the two sides of every triangle are greater than the remaining one, however assumed, as we have shewn. On this account he adds, *that it is necessary the first right lines remaining, to construct a triangle from three equal to them: but that it is requisite, any two, however taken, should be greater than the remainder, or there will not be a triangle from three lines equal to the given right lines.* But by this means he also destroys all the objections which are urged against the construction, and which may be perfectly dissolved by this addition. Hence, the present problem ranks among things determined, and not among such as are indetermined: For of problems as well as of theorems, some are indeterminate, but others without termination. Thus if we should simply say, *from three right lines which are equal to three given right lines, to construct a triangle*, the problem is indeterminate and impossible. But if we should add, *two of which, however assumed, are greater than the remainder*, the problem is determined and possible. For as the division of theorems takes place, according to true and false, so that of problems according to a possible and impossible enunciation. But that the objections which are urged against the construction, may be from hence dissolved, we shall learn from a little inspection: for we shall follow the words of the geometrician. Let there be three right lines, $a\,b\,c$, of which any two, however taken, are greater than the third, and let it be required to accomplish the thing proposed. Let there be placed a certain right line $d\,e$, on one part finite, as at the point d: and on the other part infinite. Then place $d\,f$ equal to a, but $f\,g$ to b: and $g\,h$ to c. And from the centre f, but interval $f\,d$, let a circle k be described. Again, with the centre g, but interval $g\,h$, let the circle l be designed [*sic.*]; and the circles will intersect each other. For this is assumed by the institutor of the Elements. But it



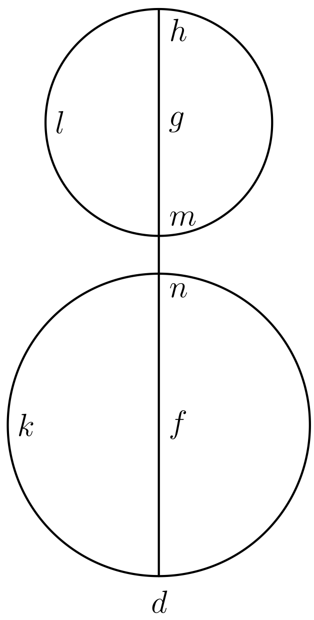
may be asked how this takes place? For perhaps they either only touch each other, or they do not even touch. Since it is necessary that they should suffer some one of three cases, I mean that they should either intersect or touch, or be distant from each other[161]. I say, therefore, that they necessarily intersect each other.

[161] [DRW:As Heath points out in his commentary on this proposition, Proclus neglects to consider all cases where the circles do not cut one another: one of the circles might be wholly contained within the other; the circles might touch internally.]

For let them in the first place, touch each other. Because, therefore, the



point f is the centre of the circle k, $d\,f$ is equal to $f\,n$. And because the point g is the centre of the circle l, $h\,g$ is equal to $g\,m$. The two, therefore, $d\,f$, $g\,h$, are equal to one, viz. to $f\,g$. But they were placed greater than one, as also a, together with c, is greater than b. They are therefore equal to it, and at the same time greater, which is impossible. Again, if it be possible, let the circles be distant from each other, as k and l.



Because, therefore, the point f, is the centre of the circle k, $d\,f$, is equal to $f\,n$. And because the point g, is the centre of the circle l, $h\,g$ is equal to $g\,m$. The whole, therefore $f\,g$, is greater than the two, $d\,f$, $h\,g$: for $f\,g$, exceeds $d\,f$, $g\,h$, by $n\,m$. But it was supposed that $d\,f$, $h\,g$, were greater than $f\,g$, in the same manner as a and c are greater than b. For $d\,f$ was placed equal to a, but $f\,g$, to b, and $h\,g$ to c. It is necessary, therefore, that the circles $k\,l$, should intersect each other. Hence, the institutor of the Elements very properly receives them cutting one another: since of the three right lines, he supposes two greater than the third, however, they may be assumed, but neither equal to, nor less than one. But it is necessary that when the circles touch, two of the lines should be equal to the third; and that when they are distant from each other, two should be greater than the remainder.