The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 21

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[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 2, pp. 119–122 (1792).]

PROPOSITION XXI¹⁵⁸. THEOREM XIV.

If upon one side of a triangle, two right lines beginning from the extremities, are internally constituted, the constituted right lines will be less than the other sides of the triangle, but they will contain a greater angle.

That which is expressed by the proposition, is, indeed, manifest; and the demonstration adopted by the elementary institutor, is evident; and the theorem is consequent to the first principles, since it depends on two theorems, the one previously exhibited, and the sixteenth. For in order to shew, that the lines internally consituted, are less than the external, the theorem is required, which says, the two sides of every triangle, are greater than the remaining one: but for the purpose of confirming that the angle comprehended by them is greater than that comprehended by the external sides, that theorem procures the greatest utility, which says, the external angle of every triangle, is greater than the internal and opposite angle. But you will receive at the same time, conviction of geometrical diligence, and a commemoration of things admirable in the mathematical disciplines, if we shall shew that it is possible within a certain triangle, upon one of its sides, not upon the whole, but upon some one of its parts, to constitute two right lines greater than the external right lines¹⁵⁹; and again, others comprehending a less angle, and comprehended in the angle made by the external lines. For this being exhibited, it will at the same time be manifest, that the institutor of the Elements necessarily adds, that the internally constituted lines must begin from the extremities of the common basis; and must be constituted upon one whole side, and not upon any one of its parts: but likewise, as I have said, one of the admirable things which geometry contains, will be manifest. For is it not, indeed, admirable, that lines constituted upon the whole side, should be less than the external sides: but that those constituted upon a part should be greater? Let there be then a right angled triangle a b c, having the angle at the point b right, and take in the side bc, any point d, and connect a d. Hence, a d is greater than a b. Take from a d a part equal to a b, which let be d e, and bisect e a, in the point f, and connect f c. Because,

¹⁵⁸[DRW—Printed as "PROPOSITION XXIV" in the 1792 edition.]

¹⁵⁹Pappus in Mathem. Collect. shews that any two sides, whose ratio to the external sides is less than two to one, may be inscribed after this manner in a triangle.



therefore, a f c is a triangle, the lines a f, f c, are greater than a c. But a f is equal to f e. The right lines therefore, f e, f c, are greater than a c. But d e is equal to a b. Hence, the right lines f c, f d, are greater than the right lines a b, a c, and they are internally constituted.

Let there be again an isosceles triangle a b c, having the base b c, greater than either of the equal sides. Then from b c, cut off b d, equal to a b, and connect a d, and take in a d, any point e, and connect e c. Because, therefore,



ab, is equal to bd, the angle bad, is also equal to the angle bda. And because the angle bda is external to the triangle edc, it is greater than the internal and opposite dec. Hence, the angle bad, is greater than the angle dec. Much more, therefore, is the angle bac, greater than the angle dec; and bac is contained by external lines, but dec by internal lines. Within a triangle, therefore, right lines de, ec, comprehending a lesser angle, are constituted within the angle comprehended by the external lines; and the thing proposed is shewn without employing the parallel lines of expositors. Hence, it is necessary that the constituted right lines should begin from the extremities of the basis; for those which are constituted upon any one of its parts, are shewn to be sometimes greater than the external lines, and to comprehend a lesser angle. But when they are constituted in this manner, beginning from the extremities, the species of triangles, called (ἁxiδoɛiôῆ) or, similar to the point of a spear¹⁶⁰, presents itself to our view; and is one of the admirable things contained in geometry, viz. to find a quadrilateral triangle.

¹⁶⁰See likewise concerning these triangles. Vol. I. p. 173 of these Commentaries.

As for example, the triangle A B C. For it is contained by the four sides B A, A C, B D, D C; but it has three angles, one at B, the other at A, and the other at C. And hence, the present figure is a quadrilateral triangle.

