The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 20

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August 2020

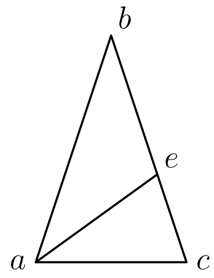
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 115–119 (1792).]

PROPOSITION XX. Theorem XIII.

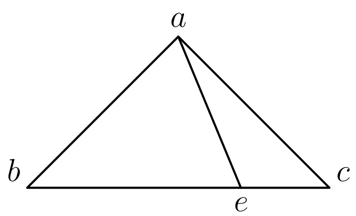
Two sides of every triangle, however taken, are greater than the remaining one.

The Epicureans oppose the present theorem, asserting that it is manifest even to an ass; and that it requires no demonstration: and besides this, that it is alike the employment of the ignorant, to consider things manifest as worthy of proof, and to assent to such as are of themselves immanifest and unknown; for he who confounds these, seems to be ignorant of the difference between demonstrable and indemonstrable. But that the present theorem is known even to an ass, they evince from hence, that grass being placed in one extremity of the sides, the ass seeking his food, wanders over one side, and not over two. Against these we reply, that the present theorem is indeed manifest to sense, but not to reason producing science: for this is the case in a variety of concerns. Thus for example, we are indubitably certain from sense, that fire warms, but it is the business of science to convince us how it warms; whether by an incorporeal power, or by corporeal sections; whether by spherical, or pyramidal particles. Again, that we are moved is evident to sense, but it is difficult to assign a rational cause how we are moved; whether over an impartible, or over an interval: but how can we run through infinite, since every magnitude is divisible in infinitum? Let, therefore, the present theorem, that the two sides of a triangle are greater than the remainder, be manifest to sense, yet it belongs to science to inform us how this is effected. And thus much may suffice against the Epicureans.

But it is requisite to relate the other demonstrations of the present theorem, such as Heron, and the familiars of Porphyry have fabricated, without producing the right line, after the manner of Euclid. Let there be a triangle $a\,b\,c$, it is requisite, therefore, to shew, that the sides $a\,b$, $a\,c$ are greater than the side $b\,c$. Bisect the angle at a, by the right line $a\,e$. Because, therefore, the angle $a\,e\,c$, is external to the triangle $a\,b\,e$, it is greater than the angle $b\,a\,e$. But the angle $b\,a\,e$, was placed equal to the angle $e\,a\,c$. The angle, therefore $a\,e\,c$, is greater than the angle $e\,a\,c$. Hence, the side also $a\,c$, is greater than the side $c\,e$. And for the same reason the side $a\,b$, is greater than the side $b\,e$. For the angle $a\,e\,b$ is external to the triangle $a\,e\,c$, and is greater than the angle $c\,a\,e$; that is than the angle $e\,a\,b$. And on this account the side $a\,b$, is greater than the side $b\,e$. The sides, therefore $a\,b$, $a\,c$, are

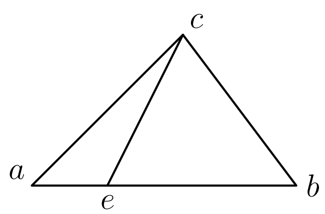


greater than the whole side $b\,c$. And the like may be shewn of the other sides. Let there again be a triangle $a\,b\,c$. If therefore the triangle $a\,b\,c$, be equilateral, two sides will be doubtless greater than the remaining one: for when there are three equal quantities, any two are double of the remainder. But if it be isosceles, it will have a base either less, or greater than each of the equal sides. If therefore the base be less, the two sides are given greater than the remainder. But if the base be greater, let it be $b\,c$, and cut off from it a part equal to either of the sides, which let be $b\,e$, and connect $a\,e$. Because,



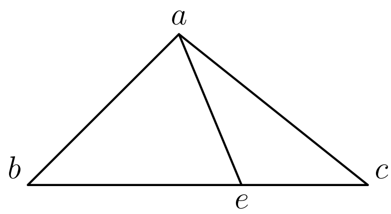
therefore, the angle $a\,e\,c$, is external to the triangle $a\,e\,b$, it is greater than the angle $b\,a\,e$. On the same account the angle $a\,e\,b$, is greater than the angle $c\,a\,e$. Hence, the angles about the point e, are greater than the whole angle about the point a, of which $b\,e\,a$ is equal to $b\,a\,e$, since $a\,b$ is equal to $b\,e$. The remainder, therefore $a\,e\,c$, is greater than the remainder $c\,a\,e$. Hence, the side $a\,c$, is greater than the side $c\,e$. But the side $a\,b$, was also equal to the side $b\,e$. The sides, therefore, $a\,b$, $a\,c$, are greater than the side $b\,c$.

But if the triangle $a\,b\,c$ be scalene, let the greatest side be $a\,b$, the middle $a\,c$, and the least $c\,b$. The greatest side, therefore, assumed with either of the others, exceeds the remainder: for by itself it is greater than either. But if we

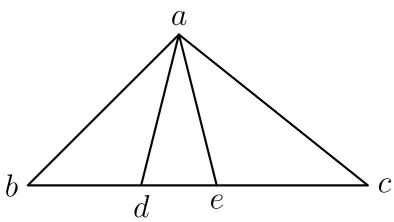


are desirous of shewing that the sides $a\,c$, $c\,b$, are greater than the greatest side $a\,b$, we must employ the same construction as in the isosceles triangle, cutting off from the greater side, a part equal to one of the other sides, and connecting the line $c\,e$, and using the external angles of the triangles.

Let there be again any triangle $a\,b\,c$. I say that the sides $a\,b$, $a\,c$ are greater than the side $b\,c$. For if they are not greater, they are either equal or less. Let them be equal, and cut off $b\,e$, equal to $a\,b$. The remainder, therefore, $e\,c$, is



equal to $a\,c$. Because then, $a\,b$, $b\,e$, are equal, they subtend equal angles; and this is likewise true of $a\,c$, $e\,c$, because they are equal. Hence, the angles at the point e are equal to the angles at the point a, which is impossible. Again let the sides $a\,b$, $a\,c$, be less than $b\,c$, and cut off $b\,d$, equal to $a\,b$, and $e\,c$ to $a\,c$. Because, therefore, $a\,b$ is equal to $b\,d$, the angle $b\,d\,a$, is not unequal to



$b\,a\,d$. And because $a\,c$ is equal to $c\,e$, the angle $c\,e\,a$, is equal to the angle $e\,a\,c$. Hence, the two angles $b\,d\,a$, $c\,e\,a$, are equal to the two $b\,a\,d$, and $e\,a\,c$. Again, because the angle $b\,d\,a$ is external to the triangle $a\,d\,c$, it is greater than the angle $e\,a\,c$: for it is greater than $c\,a\,d$. By a similar reason also, because the angle $c\,e\,a$, is external to the triangle $a\,b\,e$, it is greater than the angle $b\,a\,d$: for it is greater than the angle $b\,a\,e$. Hence, the angles $b\,d\,a$, $c\,e\,a$, are greater than the two $b\,a\,d$, $e\,a\,c$. But they were also equal to them, which is impossible. The sides, therefore, $a\,b$, $a\,c$ are neither equal to, nor less than the side $b\,c$, but greater. And the like may be exhibited in others.