The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 19

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August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 112–115 (1792).]

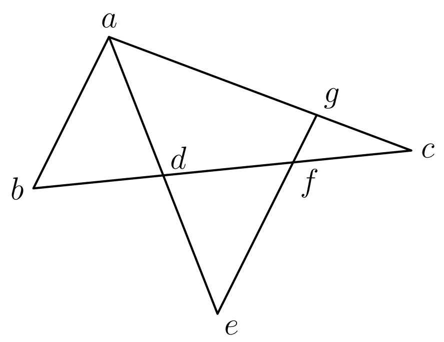
PROPOSITION XIX. Theorem XII.

The greater side of every triangle subtends the greater angle.[156]

This is the converse of the preceding theorem; and the datum, as well as the object of enquiry, is simple in each. Add too, that what was conclusion there, is hypothesis here: and what was hypothesis there is conclusion in this. But the former precedes, because it has the inequality of the sides given; and this follows, because it supposes unequal angles. For *sides*, indeed, seem to contain right-lined figures, but the *angles* appear to be contained; and the mode of demonstration in the former is ostensive, but in this it concludes the thing proposed by a deduction to an impossibility. The geometrician, therefore, by division, reasons concerning that which is impossible: for the angles being unequal: *I say*, (says he) *that the sides also subtending the unequal angles are unequal; and the greater subtends the greater given angle.* For it that which subtends the greater angle is not greater, it is either equal, or less. But if it be equal, the angles also which they subtend, are equal by the fifth. But if less, the angle also which it subtends, is less by the preceding: for it was shewn that that the greater side subtends the greater angle, and the less the lesser. But the angles have a contrary position; and hence, the one side is greater than the other.

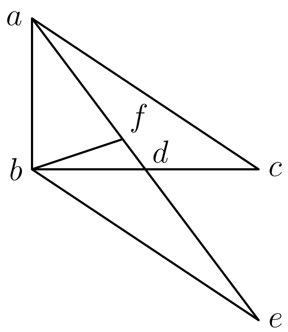
[156][DRW—The statement of Proposition 19 is here identical to that of Proposition 18. Sir Thomas L. Heath translates the statements of Proposition 18 and Proposition 19 of Euclid's *Elements* as follows: for Proposition 18, "*In any triangle the greater side subtends the greater angle*"; for Proposition 19, "*In any triangle the greater angle is subtended by the greater side.*" The Greek text of the two propositions, in Friedlein's edition of Proclus's commentary on Euclid is given as follows: for Proposition 18, Παντὸς τριγώνου ἡ μείζων πλευρὰ τὴν μείζονα γωνίαν ὑποτείνει; for Proposition 19, Παντὸς τριγώνου ὑπο τὴν μείζονα γωνίαν ἡ μείζων πλευρὰ ὑποτείνει. The Greek text of these propositions in Heiberg's edition of Euclid is identical. In Proposition 18, it is shown that if one side of a triangle is greater than another, then the angle subtended by the greater side is greater than that subtended by the lesser side. In Proposition 19, the converse is shown.]

But it is possible that we may exhibit the thing proposed, without this division. For if the angle of a triangle be bisected, and the right line drawn to the base, cutting the angle, divides it into unequal parts, the sides containing that angle will be unequal, and the greater will be that which coincides with the greater segment of the base, but the less that which coincides with the lesser. Let there be a triangle $a\,b\,c$, and let the angle at a be bisected, by the right line $a\,d$, and let $a\,d$ cut the base $b\,c$, into unequal parts, and let $c\,d$ be greater than $b\,d$. I say that the side $a\,c$ is greater than the side $a\,b$. Produce $a\,d$ to the point e, and place $d\,e$ equal to $a\,d$. And because $d\,c$ is greater than $d\,b$, place $d\,f$ equal to $b\,d$, and connect $e\,f$, and produce it to the point g. Because, therefore $a\,d$ is equal to $d\,e$, and $b\,d$ to $d\,f$, the two are equal to the



two, and they comprehend equal angles at the vertex. Hence, the base $b\,a$, is equal to the base $e\,f$, and all, therefore, are equal to all. On this account also the angle $d\,e\,f$, is equal to the angle $d\,a\,b$. But this is not unequal to $d\,a\,g$. Hence the side $a\,g$ is equal to the side $e\,g$, by the sixth. The side, therefore, $a\,c$, is greater than the side $e\,f$. But the side $f\,e$, is equal to the side $a\,b$; and hence, the side $a\,c$, is greater than the side $a\,b$, which was to be demonstrated.

This being pre-assumed, we can shew that the greater side subtends the greater angle. Let there be a triangle $a\,b\,c$, having the angle at the point b, greater than the angle at the point c. I say that the side $a\,c$, is greater than the side $a\,b$. Let $b\,c$ be bisected in the point d, and connect $a\,d$, and draw $d\,e$, equal to $a\,d$, and connect $b\,e$. Because, therefore, $b\,d$, is equal to $d\,c$,



and $a\,d$ to $d\,e$, the two are equal to the two, and they comprehend equal angles at the vertex. Hence, the base $b\,e$, is equal to the base $a\,c$, and all are equal to all. Hence too, the angle $d\,b\,e$, is equal to the angle at the point c, but less than the angle $a\,b\,d$. Let the angle, therefore $a\,b\,e$, be bisected[157] by the right line $b\,f$. Hence, $e\,f$, is greater than $f\,a$. Because, therefore, the angle at the point b, of the triangle $a\,b\,e$, is bisected by the right line $b\,f$, and $e\,f$ is greater than $f\,a$, it follows from what has been previously shewn, that the side $b\,e$, is greater than the side $b\,a$. But $b\,e$ has been shewn to be equal to $a\,c$. The side, therefore, $a\,c$, is greater than the side $a\,b$; and the object of enquiry is exhibited. And it is manifest that the institutor of the Elements, avoiding a variety of demonstration, refrains from this mode of demonstrating, and employs a method of proof, which leads from division to an impossibility, because he was willing to fabricate the converse to the preceding, without any intervening medium. For the eighth theorem, indeed, which is the converse of the fourth, brings great disturbance, because it makes conversion difficult to be known. For it is more excellent to exhibit converse theorems, by preserving the continuity through an impossible, than to destroy the continuity by a principal demonstration. And hence, Euclid shews almost all converse theorems by a deduction to an impossibility.

[157] [DRW—The text is here corrected in accordance with an erratum prefixed to the first volume of the 1792 edition.]