# The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 18

Transcribed by David R. Wilkins

August 2020

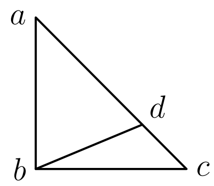
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 108–112 (1792).]

PROPOSITION XVIII. THEOREM XI.

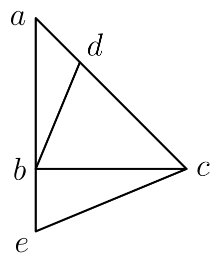
The greater side of every triangle, subtends the greater angle.

That the equality of the sides in every triangle, forms the equality of the angles which they subtend, and that in like manner the equality of the angles shews the equality of their subtending sides, we learn from the fifth and sixth theorems. But that the equality of those angles, which are subtended by the sides, follows the inequality of the sides, and the contrary we now learn by the present eighteenth and nineteenth theorems. For the one shews that the greater angle is contained under the greater side, but the other, that the greater side subtends the greater angle; because these are mutually converted, but the same symptoms are contemplated in things contrary, as in the fifth and sixth theorems. But it is manifest, that we proportionally assume the greater and less side, in scalene triangles, that we distinguish the greatest, middle, and least, and the angles in a similar manner: but in isosceles triangles, the greater and less, simply assumed, are sufficient; for there is one side which is unequal to two, because it is either greater or less, as these theorems cannot take place in equilateral triangles. And here you may observe, that the theorems which exhibit the equality of angles or sides, agree with equilateral to such as are isosceles and scalene. But the cause of this is, because of triangles, some are produced from equality alone, others from inequality alone, and others from the conjuntion of both, which are partly constituted from equality, and partly through inequality. And some are allied to *bound*, others to *infinity*, and others are generated from the mixture of both. Hence the ternary permeates through all geometrical forms, as through lines, angles, and figures; and among figures, through such as are trilateral, quadrilateral, and all the rest in a consequent order. But *bound*, likewise, must be considered as inherent in geometrical forms, as well through similitude, as equality; and *infinite*, both by dissimilitude and inequality; and that which is mixt, sometimes from the union of similitudes, and dissimilitudes, and sometimes from the union of equalities, and inequalities. But the reason of this also, is because geometrical forms regard both quantity and quality. And we have assigned these, because, when we have determined these two, it will be manifest to us, that when the institutor of the Elements says, *of every triangle*, he does not also speak of the equilateral, but of that which has a greater and less side: for it is necessary to consider the object of enquiry, as consequent to the preceding datum; and that the triangle which has a greater and less side, contains a greater angle, under the greater side.

But because the geometrician, when in the construction he receives the triangle $a\,b\,c$, and the side $a\,c$, greater than the side $a\,b$, in order that he may show, that the angle at the point $b$ is greater than at the point $c$, from the side $a\,c$, he cuts off a right line $a\,d$, equal to the side $a\,b$: on this account it



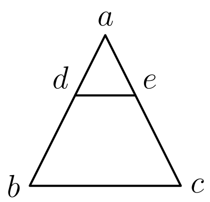

may be said that it is necessary to make the ablation[154] at the point $c$, let us therefore exhibit the thing proposed upon this hypothesis, according to Porphyry, as follows. Let $d\,c$ be equal to $a\,b$, and produce $a\,b$ to the point $e$, and place $b\,e$ equal to $d\,a$. The whole, therefore, $a\,e$, is equal to the whole $a\,c$. Connect $e\,c$. Because, therefore $a\,e$ is equal to $a\,c$, the angle, also, $a\,e\,c$,



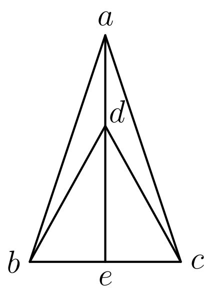

is equal to the angle $a\,c\,e$, (by the fifth). Hence, the angle $a\,e\,c$ is greater than the angle $a\,c\,b$. But the angle, also, $a\,b\,c$ is greater than the angle $a\,e\,c$; because one side of the triangle $c\,b\,e$ is produced, viz. $b\,e$, and so the angle $a\,b\,c$, since it is external, is greater than the internal and opposite angle. Much more, therefore, is the angle $a\,b\,c$, greater than the angle $a\,c\,b$, which was to be shewn. And such are the geometrical exhibitions of the present theorem.

[154][DRW—Thomas Taylor employs the word "ablation" to translate the Greek noun αφαίρεσις. This word signifies *taking away* or *removal*, and the word *ablation* employed by Taylor clearly derives from the past participle of the Latin verb *abferre*, which, being compounded of the preposition *ab* and the common Latin verb *ferre* (which has principal parts *fero*, *ferre*, *tuli*, *latum*), would also signify *to carry or bear away.* The term *ablation* is used in medical contexts to signify operations for the removal of tissue etc. from body organs such as the heart.]

But it is manifest that the cause of this symptom is the amplification, or diminution according to magnitude, of the side subtending the angle. For when it is greater, it more amplifies the angle; but when less, at the same time it diminishes, and gives a greater contraction to the angle. And this takes place on account of the right line being situated in its extremities: for through its being placed in its extremities, it changes likewise the magnitudes of the angles, according to the increase and decrease which it receives. And this we affirm in one triangle, since it is possible that the same angle may be subtended by a greater or less right line; and that the same right line may subtend a greater and less angle. For let the triangle happen to be an isosceles one, $a\,b\,c$, and let there be taken in the side $a\,b$, a point[155] $d$, and let $a\,e$ be taken equal to $a\,d$, and connect $d\,e$. The right lines therefore, $d\,e$, $b\,c$,




subtend the angle at the point $a$, of which the one is greater, but the other less. And in the same manner infinite right lines, greater and less subtending the angle $a$. Again, let the triangle $a\,b\,c$ be isosceles, and let $b\,c$ be less than $b\,a$, $a\,c$, and construct on $b\,c$, an equilateral triangle $b\,d\,c$, and connect $a\,d$, and produce it to $e$. Because, therefore, the angle $b\,d\,e$, of the triangle $a\,b\,d$,




is external, it is greater than the angle $b\,a\,d$. In like manner the angle $c\,d\,e$, is greater than the angle $c\,a\,d$. The whole, therefore, $b\,d\,c$, is greater than the whole $b\,a\,c$, and the same right line subtends both, viz. the greater and the less angle. But it is shewn, that likewise greater and less right lines subtend

[155] [DRW—The translation by Thomas Taylor here reads "and let there be taken in the side $a\,b$, a point $a$." This has been changed to accord with sense and the accompanying diagram. The Greek text of Proclus, in Friedlein's edition, reads thus: καὶ εἰλήφθω ἐπὶ τῆς $\overline{\alpha\beta}$ τὸ $\overline{\delta}$ σημεῖον,]

the same angle. But in one and the same triangle, one right line subtends one angle, and the greater always the greater, and the less always the less, the cause of which we have contemplated.