The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 17

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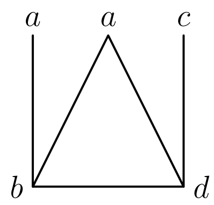
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 105–107 (1792).]

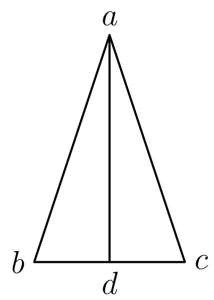
PROPOSITION XVII. Theorem X.

The two angles of every triangle, taken all possible ways, are less than two right.

In the present theorem he shews indeterminately, that any two angles of a triangle, are less than two right, but in the following theorems he determines how much they are less, and that they are deficient by the remaining angle of the triangle: for its three angles are equal to two right; and on this account the two remaining angles are less than two right. And, indeed, the demonstration of the elementary institutor proceeds in a manifest order; for it uses the preceding theorem. But it is necessary, as in the last proposition, by regarding the origin of triangles, to find the cause of the present symptom. Let then the right lines $a\,b$, $c\,d$, be at right angles to $b\,d$. It these lines



then are to form a triangle, it is requisite they should incline to each other. But their inclination diminishes the internal angles, on which account they become less than two right: for they were right before their inclination. In like manner, if we conceive right lines standing at right angles, on the side $a\,b$, the same consequences will ensue respecting the inclination of the right lines; and the angles at the points $a\,b$, will be less than two right; and so of the other side. This then is the cause of the proposition, and not the external angle being greater than either of the internal, and opposite angles: since it is not necessary that the side should be produced, nor that any angle should be constituted external to the triangle; but it is necessary that any two of the internal angles should be less than two right. Hence, it is necessary, as I have said, that the cause of this theorem should be the inclination of the right lines diminishing the angles at the base. But as the institutor of the elements exhibits the object of enquiry, by the external angle, we may accomplish this, without producing any one of the sides. Thus let there be a triangle $a\,b\,c$, and let there be taken in the side $b\,c$, any point d, and let $a\,d$



be connected. Because, therefore, one side of the triangle $a\,b\,d$, is produced, viz. $b\,d$, the external angle $a\,d\,c$, is greater than the internal $a\,b\,d$. Again, because one side of the triangle $a\,d\,c$ is produced, viz. $c\,d$, the external angle $a\,d\,b$, is greater than the internal $a\,c\,d$. But the angles about the right line $a\,d$, are equal to two right, by the thirteenth of this. Hence, the angles $a\,b\,c$, $a\,c\,b$ are less than two right. In like manner, we may shew, that the angles $b\,a\,c$, and $b\,c\,a$, are less than two right, by taking a point in the side $a\,c$, and by connecting the point b with the assumed point. And again, we may affirm, that the angles $c\,a\,b$, $a\,b\,c$, are less than two right, by taking a point in the side $a\,b$, and by connecting a right line, from the point c, and the received point. And thus the thing proposed, is exhibited by the same theorem, without producing any side of the triangle. Hence, it is possible, that by this, the theorem may be proved, which asserts, *that, from the same point, two perpendiculars cannot be drawn, to one right line.* For let there be drawn, if possible, from the point a, two perpendiculars $a\,b$, $a\,c$, to the right line $b\,c$. Then the angles $a\,b\,c$, $a\,c\,b$, are right. But because $a\,b\,c$ is a triangle, two of its angles are less than two right. The angles, therefore, $a\,b\,c$, $a\,c\,b$, are less than two right. But they are also equal to two right, because they are perpendiculars, which is impossible. Hence, from the same point, to the same right line, two perpendiculars cannot be drawn.