The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 16

Transcribed by David R. Wilkins

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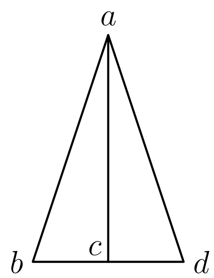
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 102–105 (1792).]

PROPOSITION XVI. Theorem IX.

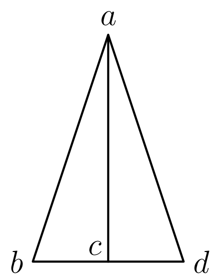
In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles.

Those who enunciated this proposition, and at the same time omitted the particle, *having one side produced*, perhaps afforded an occasion of objection to many others, as well as to Philip, (according to the narration of the mechanist Heron.) But such as were desirous of entirely removing this calumny, enunciated the theorem, with the proposed addition, corresponding with the general manner of the geometrician. For in the fifth theorem, being desirous to shew, that the angles under the base of an isosceles triangle are equal, he adds, that when equal right lines are produced, the angles under the base are equal. Hence we infer, that though this proposition might be defective and imperfect in various copies, yet it was perfect and written entire, by the institutor of the Elements. What then does the proposition assert? *That in every triangle, if you produce one of its sides, you will find the angle constituted external to the triangle, greater than either of the internal and opposite angles.* For a little after, this angle will be shewn equal to both, but it is proved to be greater than either in the present; and he necessarily compares it with the opposite angles, and not with the *successive* angle. For to this last it may be both equal and less: but it is greater than either of the former. This, if this triangle should be right angled, and you conceive one of the sides comprehending the right angle to be produced, the external will be equal to the successive angle. But if it should happen to be obtuse-angled, the internal angle may be greater than the external; and it is on this account that he does not compare the external with the successive angle, but with the opposite angles. For of the angles within a triangle, the successive angle borders on the external, but the two others are opposite. Hence, the external angle is greater than either of the successive, but may not exceed the successive angle to which it is proximate. But some conjoining these two theorems, I mean the present, and the following, enunciate the proposition thus. *In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles; and any two of the internal angles, are less than two right.* But there is occasion for the connection of these theorems, because the geometrician himself, a little after, enunciates the proposition after this manner, in equal angles, for he says: *In any triangle having one of its sides produced, the external angle is equal to the two interior, and opposite angles; and the three internal angles of a triangle are equal to two right.* Hence, they think it proper in the present similar case, to connect the objects of investigation, and to make the proposition a composite. But if the datum be enunciated with this addition, it also will be a composite, (since it is requisite to understand two things, viz. the subject triangle, and one side produced:) and if the datum be given without this, it will be a *composite* in capacity, but *simple* in energy; for this must be received at the same time as a datum; since while we suppose an external angle, we must pre-suppose the side as produced.

But we may assume from the present theorem, that it is impossible from the same point, for three equal right lines to fall on the same right line. Thus let there be drawn from one point a, three equal lines $a\,b$, $a\,c$, $a\,d$, to the right line $b\,d$. Because, therefore, $a\,b$ is equal to $a\,c$, the angles at the base



are equal. Hence, the angle $a\,b\,c$, is equal to the angle $a\,c\,b$. Again, because $a\,b$ is equal to $a\,d$, the angle $a\,b\,d$, is equal to the angle $a\,d\,b$. But the angle $a\,b\,c$, was equal to the angle $a\,c\,b$. Hence, the angle $a\,c\,b$, is equal to the angle $a\,d\,b$, the external, to the internal and opposite, which is impossible. From the same point therefore, to the same right line, three equal right lines cannot be drawn. But by the present theorem, we can also demonstrate, that if a right line falling on two right lines, makes the external angle equal to the internal and opposite, those right lines will by no means make a triangle, nor coincide, because the same thing would be both greater and equal, which is impossible. Thus for example, let $a\,b$, $c\,d$, be right lines, and let the right line $e\,b$ falling on them make the equal angles $a\,b\,d$, $c\,d\,e$, the right lines $a\,b$, $c\,d$, will not coincide. For if they coincide, the equal angles remaining, the angle $c\,d\,e$, will be equal to the angle $a\,b\,d$. And since it is external, it will be greater than the internal and opposite angle. Hence, it is necessary, if they coincide, that the angles remain no longer equal, but that the angle at the point d, be augmented. For whether $a\,b$ remaining immoveable, we conceive that $c\,d$ is moved towards it, so as to coincide in the point c, we shall produce a greater



distance in the angle $c\,d\,e$; since $c\,d$ approaches $a\,b$, in the same proportion as it recedes from $d\,e$. Or whether $c\,d$, abiding, we conceive that $a\,b$ is moved towards it, in a similar manner, we shall by this means diminish the angle $a\,b\,d$; for it is at the same time carried towards $c\,d$, and to $b\,d$. Or whether we conceive both of them tending to each other, we shall find that $a\,b$ by tending to $c\,d$, contracts the angle $a\,b\,e$; and that $c\,d$, by receding from $d\,e$, on account of the motion to the line $a\,b$, increases the angle $c\,d\,e$. Hence, it is necessary, if it be a triangle, and if the right lines $a\,b$, $c\,d$, coincide, that the external angle must be also greater than the internal and opposite angle. For either the internal angle remaining, the external is increased, or the external abiding, the internal is diminished, or the internal is contracted, and the external is more dilated. But the cause of these consequences is the motion of the right lines, the one tending to those parts where it increases the external. And from this the reader should consider, how the origin of things produces the true causes of enquiries, which we have previously surveyed.