The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 14

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 92–96 (1792).]

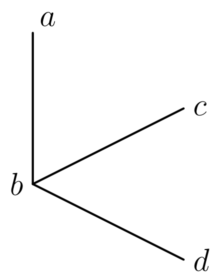
PROPOSITION XIV. Theorem VII[151].

> If to any right line, and at a point in it, two right lines being placed in a consequent order, and not towards the same parts, make the consecutive angles equal to two right, those right lines shall be in a direct position to each other.

The present theorem is the converse of the foregoing: for such as are converse are always consequent to preceding theorems. Since, therefore, the former had constituted a right line upon a right line, and had shewn that it made the successive angles either two right, or equal to two right; in the present theorem he receives the equality of the angles to two right, which are formed at some right line, but he shews that it is one right line which produces their equality. Hence, that which was a *datum* in the former, is in the present theorem an object of enquiry; and it shewn by a deduction to an impossibility. For after this manner the converse of theorems ought to be exhibited; but in problems they should receive principal demonstrations. But in this theorem we may also perceive the greatest and most admirable diligence of this proposition producing science. For in the first place, after he had said, *if to any right line*, he adds, *and at a point in it*; for what if the two extremes of the right line existing, one of the right lines should be drawn from the one extreme, but the other from the remaining one, and should form angles at the right line, equal to two right, would they on this account have a direct position? And how can this take place in lines drawn from different points of the right line? It is on this account also, that he adds, *and at a point in it*, since he is willing that both should be in the same point. But in the second place, because it is possible that the right lines which are drawn, may be at the same point, and not consequent (since we may receive infinite right lines placed at the same point) he adds the particle, *in a consequent order*. And in the third place, because the word *consequent* may be considered as well at the same parts as on both sides: but because it is impossible that lines which are consequent at the same parts should be mutually in a direct position, this indeed he explains, but affords us an opportunity of considering that consequent right lines are to be received in position on both sides; since

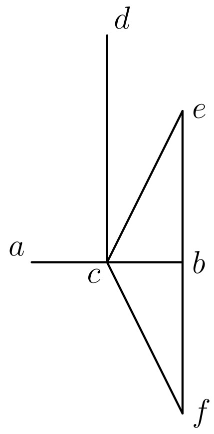
[151] [DRW—Printed VI in the 1792 publication]

these also can be shewn to be in a right line. Let there be placed at the right line $a\,b$, and at a point in it b, towards the same parts, two right lines $b\,c$, $b\,d$, these therefore shall be consequent to each other. For no other right line



is situated between them. But those things are *successive*, between which there is nothing similar. Thus we call the columns *consequent*, between which there is no other column: for though the air intervenes, yet nothing of the same kind is situated in the middle. Because, therefore, they lie towards the same parts, they are by no means in a direct position, although they form two angles equal to two right; I mean the angles at the point b. For nothing hinders but that the angle $a\,b\,d$, may contain in itself, one right, and a third part of a right angle: and that the angle $a\,b\,c$, may be two thirds of a right angle. And thus much concerning the proposition.

But one petition is employed in the construction, viz. the second, which begs *to produce a right line straight forwards*, as in the demonstration he uses the preceding theorem, and two axioms; i.e. the one which says, *things equal to the same, are equal to one another*; and last the one which affirms, *that if from equal things equals are taken away, the remainders shall be equal.* But at the collection of the impossibility, he employs the axiom, which says, *the whole is greater than its part.* For it is equal one common angle being taken away, which is impossible. But that it is possible to the same right line, and at a point in it, two right lines in a consequent position, and yet, towards the same parts, may form angles belonging to that one right line, equal to two right, we may shew with Porphyry, as follows. Let there be a certain right line $a\,b$, and any point in it c, and let $c\,d$, be raised at right angles to $a\,b$, and let the angles $d\,c\,b$ be bisected by the line $c\,e$. Then from the point e, to the line $a\,b$, let there be drawn the perpendicular $e\,b$, and let $e\,b$ be produced, and place $f\,b$ equal to $e\,b$, and connect $c\,f$. Because, therefore $e\,b$ is equal to $b\,f$, but $b\,c$ is common, and they contain equal angles (for they are right), hence the base $e\,c$, is equal to the base $c\,f$. All, therefore, are equal to all. Hence, the angle $e\,c\,b$, is equal to the angle $f\,c, b$. But the angle $e\,c\,b$ is the half of a right angle: because the right angle $d\,c\,b$ was bisected by the line $e\,c$. Hence, also the angle $f\,c\,b$, is the half of one right. The angle, therefore $d\,c\,f$,



is equal to one right, and the half of a right angle. But the angle $d\,c\,e$, also, is the half of a right angle. Hence to the right line $c\,d$, and to a point in it c, two right lines are consequently placed towards the same parts, viz. $c\,e$ and $c\,f$, forming angles equal to two right, $c\,e$ causing the half of a right angle, and $c\,f$ one and a half. Lest, therefore, we should enquire after things impossible to be effected, viz. how the right lines $c\,e$, $c\,f$, forming angles at the right line $d\,c$, equal to two right, can be in a direct position to one another, the Geometrician adds the particle *not towards the same parts*. it is requisite, therefore, that the right lines which form angles equal to two right, should be placed on both sides of the right line, being raised, indeed, from one point, but drawn to different parts of the right line.