The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 12

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 85–90 (1792).]

PROPOSITION XII. Problem VII.

Upon a given infinite[150] right line, and from a given point which is not in that line, to let fall a perpendicular.

Oenopides first investigated this problem, believing it useful for astrological purposes. But he calls a perpendicular, after the manner of the ancients, a gnomon, because a gnomon, also, is at right angles to the horizon, but the same line is at right angles with a perpendicular, form which it differs only in habitude, since, as he observes a gnomon has the same subject with a perpendicular. But again, a perpendicular is two-fold, that is, it is either plane or solid. Hence, when the point from which the perpendicular right line is drawn, is in the same plane, the perpendicular is called plane; but when the point is on high, and external to the subject plane, it is called solid. And the plane perpendicular, indeed, is drawn to a right line: but the solid to a plane. Hence, it is necessary, that this last should not only form right angles, with one right line, but with all right lines in the same plane. For the perpendicular is let fall on a plane. In the present problem, therefore, the institutor of the Elements proposes to let fall a plane perpendicular. For the deduction is proposed to a right line, and the discourse, so far as all are supposed to be in the same plane. Hence, in the line at right angles we do not require infinity, because the problem is supposed to be in that right line. But in the present problem, respecting a perpendicular, he supposes the given right line infinite, because the point from which the perpendicular is to be drawn is placed external to the right line. For if it was nto infinite , the point might be received externally, and yet in a direct position, so that the protracted right line would fall upon it, and the problem not succeed. Hence, he places the right line infinite, so that the point may be received at either of its parts;

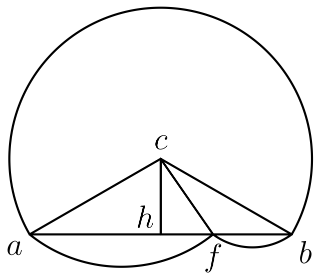
[150]Mr Simson having a great objection to the word infinite, though it is adopted by Euclid, substitutes in its place the word *unlimited*; but not in my opinion with any success. For if by *unlimited*, he means infinite, the alteration is ridiculous; but if he means only *indefinite*, or a line which has boundaries, though they are not ascertained, the problem will not succeed, as the ensuing commentary most beautifully evinces. I only add, that the reader, if he be a man of taste, and possesses any spark of the philosophic genius, must be greatly delighted with the digression of Proclus in this comment, concerning the nature of infinite, as it is perfectly philosophical and truly sublime.

and that no place may be left, in which it can be in the same direction with the given right line, unless it is in the line, and has not an external position. And on this account the right line to which the perpendicular is to be drawn is considered as infinite.

But in what manner infinite can subsist, is a matter well worthy our contemplation. For it is manifest that a right line existing infinite, a plane also will be infinite, and this in energy, if the thing proposed by Euclid be true. That among sensible particulars, therefore, there can be no magnitude infinite, according to any distance, both the dæmoniacal Aristotle, and those who received their philosophy from him, have abundantly shewn. For neither that which is moved circularly, nor any other simple body can be infinite; since the place of each is limited. But neither in separate and impartible reasons is an infinite of this kind possible. For if they neither contain dimension, nor magnitude, much less can they contain infinite magnitude. It remains, therefore, that infinite can alone subsist in the phanrasy, which at the same time the phantasy does not comprehend. For as soon as it understands, it induces form and bound to that which is understood, stops the transit of the phantasm by its intellection, pursues its progress, and infolds it in its shadowy embrase. The phantasy, therefore, is not infinite by intellection, but rather by advancing infinitely about that which is understood; and calling whatever it leaves innumerable, and incomprehensible by intelligence, infinite. For as the sight by not seeing understands darkness; so the phantasy by not understanding perceives infinite. Hence it pursues the progress of the infinite, because it is endued with an impartial power, capable of perpetually advancing: but it understands as if stopping in its progression, because infinite surpasses its comprehension. For it calls that infinite, which it leaves as undable to pass over in its pursuit. On that account when we place a given infinite line in the phantasy, in the same manner as we establish all other geometrical species, viz. triangles, circles, angles, lines, and all of this kind, we must not wonder how a line is infinite in energy, and how advancing infinitely, it applies itself to finite intellections. But cogitation, in which reasons and demonstrations reside, does not use infinite for the purpose of science, since infinite is by no means perceptible by science, but receiving it from hypothesis, it employs finite alone in its demonstrations, and assumes infinite not for the sake of infinite, but of that which is bounded and finite. For if we should grant to cogitation, that the given point, neither lies in a right line with the given finite right line, nor yet is so distant from it, that no part of the right line is subjected to the point, we shall no longer require an infinite line. That cogitation, therefore, when employing a right line, may use it without controversy and reproof, she supposes it to be infinite; and employs the infinity of the phantasy, as the foundation of infinite generation.

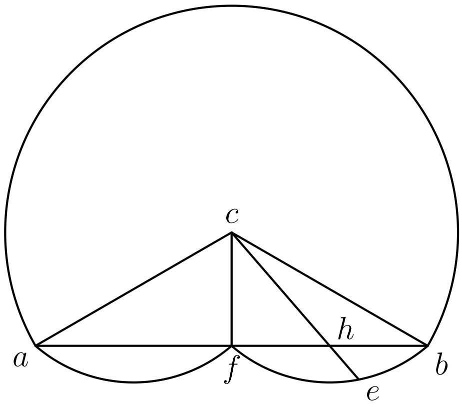
And thus much may suffice for the present concerning the nature of infinite.

But it is now requisite that we should consider the objections which are urged against the construction of this this problem. Let there be received, say they, an infinite right line ab, and let the given point be received, say they, an infinite right line ab, and let the given point be c, from which it is required to let fall a perpendicular, and let d be a point on the other side, according to the geometrician. But the circle which cuts the right line ab, in



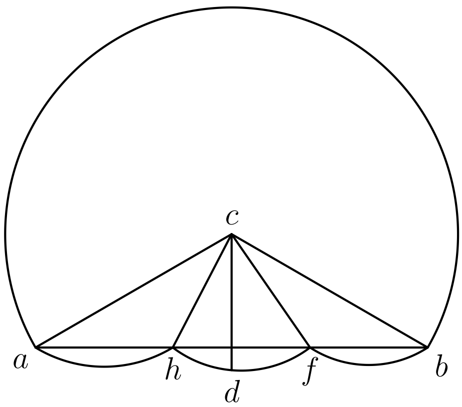
the points a and b, will cut it also in f, and will have a situation according to the figure. In answer to this, we must say, that it affirms an impossible case. For let the right line ab be bisected in h, and let ch be connected, and produced to the circumference, to the point d, and let ca, cb, cf be connected. Because, therefore, these lines are from the centre, and ah, is equal to hb, but ch is common, all are equal to all. Hence ch forms right angles at the point h. Again, because ca, cb are equal, they form equal angles at the points a and b. But ca also, is equal to cf, on which account the angle caf, is equal to the angle cfa. In like manner the angle cbf is equal to the angle cfb. Because, therefore, the angles at the points a and b, are equal, the angle, also cfa, is equal to the angle cfb, and they are successive, and consequently right. But each of the angles at the point h is right. Hence ch is equal to cf. But cf is also equal to cd, since they are from the centre. Therefore ch is equal to cd, which is impossible. Hence, the circle does not cut the right line in any other points than a and b.

But if any one should say, that he who describes a circle will bisect ab if f, we can again shew that this is impossible. For let all be described as before, and let the right line fb, be bisected in the point h. Because, therefore af, fb, are equal, but cf common, and the base ca, is equal to the base cb, all are equal to all. Hence, the angles at the point f are right. Again, because fh is equal to hb, and ch being connected, is common, and the base cf is equal to the base cb, for they are from the centre, the angles at the point h are right; for they are equal and successive. Because, therefore, each of the angles cfh, chf is right, cf is equal to ch. But cf is equal to ce, for they are from the centre, and hence ch is not unequal to ce, which is not



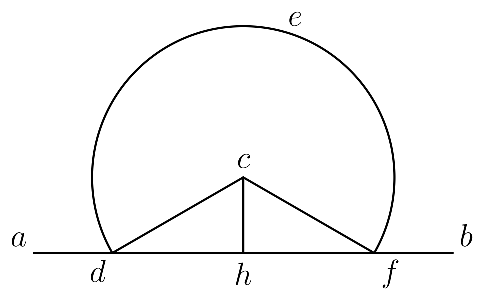
impossible.

It now remains that we run over the third objection. For the circle which is described (say they) will cut the right line in the points a, b, and in the points f, h. We therefore bisecting the right line $a\,b$ in the and connecting

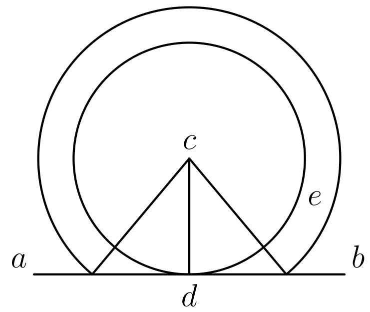


the lines $c\,a$, $c\,f$, $c\,k$, $c\,b$, can shew that this is impossible. For since $k\,a$, $k\,b$, are equal, and $c\,k$ is common, and the bases $c\,a$, $c\,b$, are equal, hence the angles at the points a and b are equal, and those at the point k right. But each of the lines is equal to $c\,f$; and hence, the angles at the point f, are right; for they are equal, because successive. Therefore $c\,f$ is equal to $c\,k$: for they subtend right angles. But $c\,f$ is equal to $c\,k$, which is impossible. Hence then, it is impossible that the circle which is described should cut the line $a\,b$ in one, two, or in more points than $a\,b$. And such are the objections against the present problem.

But there are also cases of the construction of this problem, which are to be distinguished from the objections. For cse is not the same with object; since the former shews the same thing differently, but the latter leads the objection to an inconvenience. But other expositors, not distinguishing these from one another, bring all into the same, so that it is uncertain, whether they enunciate to us in their writings, cases, or objections. We therefore distinguishing these, having enumerated the objections, shall now describe the cases of the problem. Let there be then an infinite right line $a\,b$, and a given point c. Now it may be said that there is no farther place in the other



part of the perpendicular right line, but in that only where the point c lies. Taking, therefore, in the right line $a\,b$ a point d, with the centre c, and interval $c\,d$, let us describe the circumference of a circle $d\,e\,f$, and bisecting $d\,f$ in h, let us connect the lines $c\,d$, $c\,h$, $c\,f$. Because, therefore, $d\,h$ is equal to $h\,f$, but $c\,h$ is common, and $c\,d$ is equal to $c\,f$, (for they are from the centre,) hence, the successive angles at the point h, are equal. They are, therefore, right. And hence, $c\,h$ is a perpendicular to $d\,f$. But if any one should also say that the described circle does not cut the right line $a\,b$, but touch it as the circle $d\,e$, but taking the point e externally, and using the centre c, and interval $c\,e$, as in the preceding, we shall obtain the object of our enquiry. And thus much we have said concerning the cases of the problem, for the



sake of exercising the attention of the reader.

But if we are desirous of adding contemplation likewise to these two problems, a right line erected at right angles, seems to intimate a life tending on high from inferior concerns, ascending purely, and without contamination, and abiding inflexibly with regard to natures subordinate to its own. But a perpendicular is the image of a life perpendicularly descending, and the least of all replete with generative infinity. For a right line is the symbol of an energy inflexible, and restrained in the comprehension of equality, bound, and finite. From whence, indeed, Timæus also calls the *other circle* in the divine soul, possessing the reasons of sensible natures, *right*; for in our souls it is bent with flexions of every kind, and suffers various contortions and perturbations from the unceasing whirls of generation: but among *wholes* it resides immaculate, uncontaminated, firm, and indeclinable, prior to sensible forms. But if likewise an infinite right line is the symbol of the whole of generation, which is moved infinitely and indeterminately, and besides this, of matter itself, which is deprived of bound and form: and if a point placed externally bears an image of an essence impartible, and separate from material natures, doubtless the deduced perpendicular will imitate that life which proceeds into generation with an undefiled progress from unity, and an impartible essence. But if a perpendicular cannot be shewn without circles, this also will be the symbol of an inflexibility inherent in life, through the medium of intellect. For life, indeed, since it subsists by itself as motion, is indeterminate: but it becomes terminated, and is filled with a pure and immaculate power, by participating and adhering to the circulations of intellect.