The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 11

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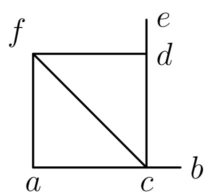
August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 80–82 (1792).]

PROPOSITION XI. Problem VI.

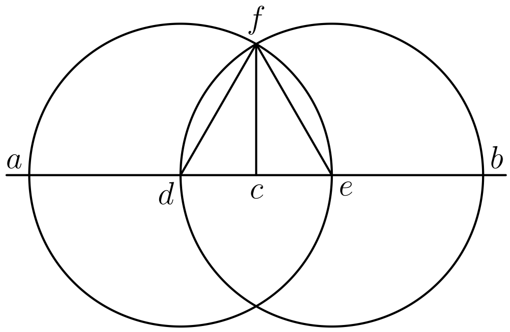
To raise a right line at right angles, to a given right line, from a given point in that line.

Whether we receive a right line on both sides finite, or on both sides infinite, or on one side infinite but on the other finite, and a point in it, the construction of the present problem will conveniently succeed to the geometrician. For though the given point should be on the extremity of the right line, by producing it we can accomplish our purpose. But it is manifest that the point in the present problem is given in *position*, since it can only be placed in position in a right line. But the right line is given according to *form*; since its magnitude is not distinguished either by proportion or position. Hence, the institutor of the Elements, employing the first and third problem, together with the eighth proposition, and the tenth definition, exhibits the thing proposed. But if any placing the point on the extremity of the right line, should ask us without producing the line, to erect upon this a line a right angles, we can likewise shew that this is possible to be effected. For let there be a right line $a\,b$, and a given point in it a, and let there be assumed in the line $a\,b$, any point c, and from this (as the present element teaches us) let a right line $c\,e$ be erected at right angles to $a\,b$. Then from $c\,e$, let $c\,d$ be taken equal to $a\,c$, and let the angle at the point c be bisected by the line $c\,f$, and at the point d let a right line be erected at right angles, coinciding with $f\,c$ in f; and lastly from the point f, to the point a, let $f\,a$ be connected. I say that the angle at the point a is right. For since $d\,c$ is equal to $c\,a$, but



$c\,f$ is common, and contains equal angles, (for the angle at the point c was bisected) hence, $d\,f$ is equal to $f\,a$, and all in like manner (by the fourth) are equal to all. The angle, therefore, at the point a, is equal to the angle at d. But the angle at the point d is right; and so consequently is the angle at a. And thus the thing required is effected. But the institutor of the Elements was not indigent of any such artifice: for he commands us to raise a line at right angles, but not at one right. It is requisite, therefore, not to receive the point in the extremity of the right line, because the perpendicular line forms angles with its subject right line, but not one angle alone.

But Apollonius raises a perpendicular as follows. Let the given right line, says he, be $a\,b$, and a given point in it c, but let there be assumed in $a\,c$ any point d, and from $c\,b$, take away $c\,e$, equal to $c\,d$. Then with the centre d,



but interval $d\,e$, let a circle be described; and again with the centre e, but interval $e\,d$, let another circle be described, and let a right line be drawn from f to c. I say that $f\,c$ is a perpendicular. For if $f\,d$, $f\,e$, are connected, they shall be equal. But $d\,c$, $c\,e$, are equal, and $f\,c$ is common. Hence, also, the angles at the point c (by the eighth) are equal. They are therefore right. And here, is it not again obvious, that this demonstration is more various than that of Euclid, and requires the description of circles, that by this means an equilateral triangle may be described upon $d\,e$, and the problem exhibited? For all the rest are common to the demonstrations. But the demonstration by a semicircle is not worthy to be remembered, since it supposes many things which are afterwards exhibited, and entirely falls from the order of an elementary institution.