The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 10

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 77–80 (1792).]

PROPOSITION X. PROBLEM V.

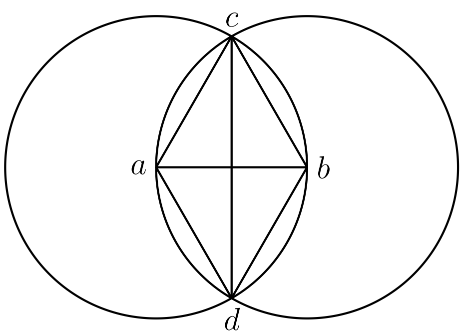
To bisect a given finite right line.

This, also, is a problem which supposes a finite right line, since we cannot terminate a line on both sides infinite. But the section of a line infinite on one side only, wherever the point is assumed, is made in unequal parts. For that part of the section which takes place on the infinite side, is necessarily greater than the remainder, because finite. Hence, the line required to be bisected, must be necessarily both ways finite. But perhaps, some excited by this problem, may think, that the doctrine of a line, not being composed from impartibles[149], is only previously received by geometricians as an hypothesis. For if it consists from impartibles, it either becomes finite, and receives its completion from *odd*, or from *even* parts. But if from such as are *odd*, it will appear that an impartible also may be cut, while a right line is bisected. And if from such as are even, the section will be unequal, because, one part, as composed from more impartibles, will be greater than the remainder. It is therefore impossible to bisect a given right line, if magnitude consists from impartibles. But if it be not composed from impartibles, it may be divided in infinitum. It appears, therefore, (say they) to be received by common consent, and to be a geometrical principle, that magnitude is among the number of things infinitely divisible. Against these we reply in the words of Geminus, that geometricians previously receive according to a common conception, that continued quantity is divisible. For we call that continuous, which is composed from conjoined parts, and this it is in every respect possible to divide. But that continued quantity may be infinitely divided, they do not previously assume, but demonstrate from proper principles. For when they shew that incommensurability is found in magnitudes, and that all are not commensurable with each other, what else can we say

[149][DRW: Thomas Taylor uses the term "impartible" in contexts where 'indivisible" is more normally employed. He explains his reasons in the preface to Volume 1 of his translation of these commentaries of Proclus (Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, two volumes, 1792): "[I] must beg leave to solicit the reader's indulgence for using the words *partible* and *impartible*, differently from their common signification. These words I have generally employed to express the meaning of μεριζος, and αμεριζὸς in the Greek, as I do not conceive that the words *divisible* and *indivisible* always convey their full signification." In footnotes, Taylor translates these Greek words μεριζος, and αμεριζὸς as "capable of parts" and "not capable of parts" respectively.]

they evince by this means, except this, that every magnitude may be divided into parts always divisible, and that we can never arrive at an impartible, by the most unwearied analysis, since this minimum would be the common measure of all magnitudes? This then is demonstrable, but that which says, *every thing continuous is divisible*, is an axiom. Hence, since a finite line also is continuous, it is divisible. And from this conception the institutor of the Elements cuts a finite right line into equal parts, but not as pre-assuming, that it is divisible in infinitum. For to be merely divisible, and to be infinitely divisible is not the same.

But the discourse of Zenocrates inferring indivisible lines, is confuted by this problem. For if it be a line, it is either right, and may be bisected; or circular, and it is greater than a certain right line; (since every circular has a certain right line less than itself); or it is mixt, and on this account is the more divisible, since composed from simple divisible lines. But this must be deferred to some posterior speculation. However, the geometrician bisects a finite right line, employing in the construction the first and ninth propositions; but using in the demonstration the fourth alone; for by the angles he shews the equality of the bases. But Apollonius Pergæus bisects a given finite right line after the following manner. Let there be (says he) a finite right line $a\,b$, which we are required to bisect, and with the centre a, but interval $a\,b$, let a circle be described. And again, with the centre b, but interval $b\,a$, let another circle be described, and let the right line $c\,d$, connect the common sections of the circles; this shall bisect the right line $a\,b$. For let



the equal lines $d\,a$, $d\,b$, $c\,a$, $c\,b$ be connected; these being equal, because each is equal to $a\,b$. But $c\,d$ is common, and $d\,a$ is equal to $d\,b$ on the same account. Hence the angle $a\,c\,d$, is equal to the angle $b\,c\,d$; and so (by the fourth) $a\,b$ is bisected. Such then, according to Apollonius, is the demonstration of this problem, assumed, also, from an equilateral triangle; but instead of exhibiting the bisection of the line, from the bisection of the angle at the point c, it shews this from the equality of the bases. The demonstration, therefore, of the institutor of the Elements, is much better, since it is both more simple, and emanates from principles.