The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 8

Transcribed by David R. Wilkins

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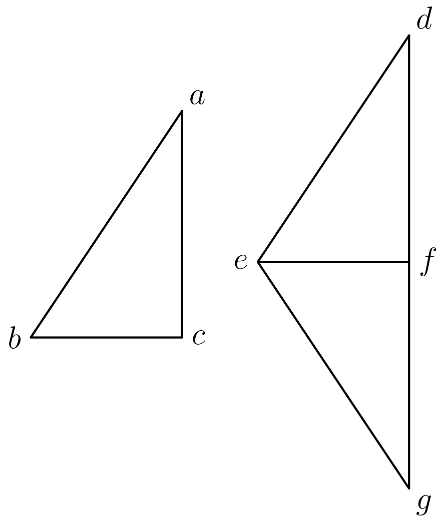
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 65–70 (1792).]

PROPOSITION VIII. Theorem V.

If two triangles have two sides equal to two, each to each, and have the base equal to the base: then the angles contained by the equal right lines, shall be equal to each other.

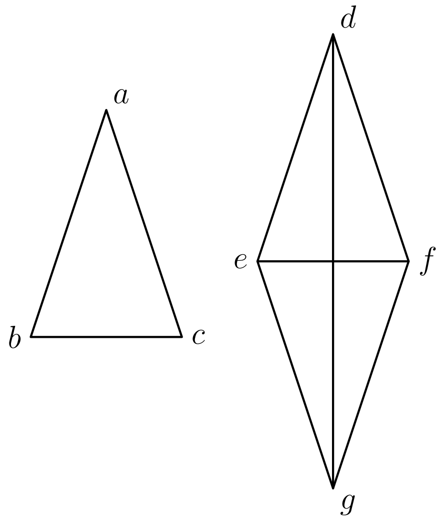
This eighth theorem is the converse of the fourth: but it is not assumed according to a principal conversion. For it does not make the whole of its hypothesis a conclusion; and the whole conclusion an hypothesis. But connecting together some part of the hypothesis of the fourth theorem, and some part of the objects of enquiry, it exhibits one of the data which it contains. For the equality of two sides to two, is in each an hypothesis; but the equality of base to base, is, in the fourth, an object of investigation, but in the present a datum; and the equality of angle to angle is, in the former, a datum, but in the latter, an object of enquiry. Hence, a change alone of data, and objects of investigation, produces conversion. But if any one desires to learn the cause why this theorem is placed in the order of the eighth proposition, and not immediately after the fourth, as its converse, in the same manner as the sixth after the fifth, of which it is the converse, since many converted propositions follow their precedents, and are exhibited after them without any intervening medium, to this we must reply, that the eighth, indeed, is indigent of the seventh proposition. For its truth is evinced by a deduction to an impossibility, but the nature of an impossible becomes known from the seventh. And, this again, in its demonstration, is indigent of the fifth. Hence, the seventh and fifth theroems were necessarily assumed, previous to the present. But because the *converse* to the fifth obtained a demonstration easy, and from *things first*, it was very properly placed after the fifth, on account of its alliance with that theorem; and because, since it is shewn by a deduction to an impossibility, it confutes that which is impossible from common conceptions, and not as the eighth from another theorem. For things opposing common conceptions, are more evident for the purpose of confutation than such as contradict theorems: since these are assumed by demonstration, but the knowledge of axioms is better than demonstration. But the institutor of the elements exhibits what is now proposed from the previously demonstrated seventh theorem.

But the familiars of Philo assert, that they can demonstrate this theorem, without being indigent of any other. For let there be conceived (say they) two triangles $a\,b\,c$, $d\,e\,f$, having two sides equal to two, and the base $b\,c$ equal to the base $e\,f$. Likewise let the bases coincide with each other; and let the two triangles $a\,b\,c$, $d\,e\,f$, be so placed in the same plane, that their vertices may be opposite, and so that $e\,f\,g$ may be the equal substitute of $a\,b\,c$. And

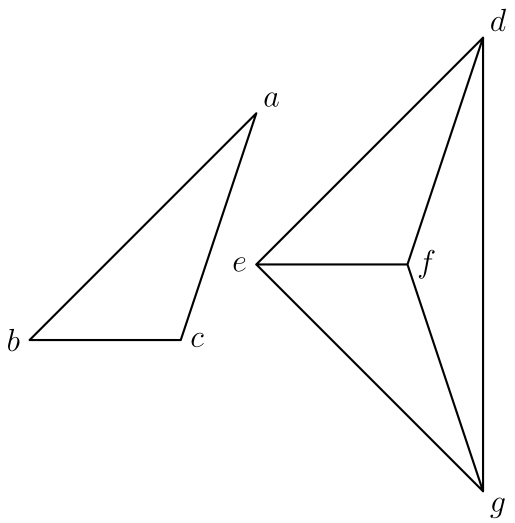


let $e\,g$ be equal to $d\,e$ but $f\,g$ to $d\,f$. Hence, $f\,g$ will either be placed in a right line with $d\,f$, or not in a right line. And if not in a right line, it will either make with it an angle according to the internal part, or according to the external. Let it first be placed in a right line. Because, therefore, $d\,e$ is equal to $e\,g$, and $d\,f\,g$ is one line, the triangle $d\,e\,g$ is isosceles, and the angle at the point d, is equal to the angle at the point g.

But if it does not lie in a right line, it will make an angle inward; and in this case let $d\,g$ be connected.



[Be]cause, therefore, $e\,d$, $e\,g$, are equal, and the base is $d\,g$, the angle $e\,d\,g$ also, is equal to the angle $e\,g\,d$. Again, because $d\,f$ is equal to $f\,g$, and the base is $d\,g$, the angle, also, $f\,d\,g$ is equal to the angle $f\,g\,d$. But the angle $e\,d\,g$ was also equal to the angle $e\,g\,d$. Hence, the whole $e\,d\,f$, is equal to the whole $f\,g\,e$, which was required to be demonstrated. But in the third place,let $f\,g$ make an angle with $d\,f$, externally, and let the right line $d\,g$ be connected. Because, therefore $d\,e$, $e\,g$, are equal,and the base is $d\,g$, the



angles $e\,d\,g$, $d\,g\,e$, are equal. Again, because $d\,f$, $f\,g$ are equal, and the base is $d\,g$, the angle $f\,d\,g$, is equal to the angle $f\,g\,d$. But the whole angles $e\,d\,g$, $d\,g\,e$, were mutually equal. Hence, the remaining angles $e\,d\,f$, $f\,g\,e$ will be equal to each other. And thus the thing proposed is invented according to any position of the right line $f\,g$, and we may demonstrate the theorem, without employing the seventh proposition.

Is, then (say they), the seventh proposition introduced in vain by the institutor of the elements? For if we only assume it on account of the eighth, but the eighth may be exhibited without it, does not the seventh appear entirely useless? To these enquiries we must reply in the words of our predecessors, that the seventh theorem, being demonstrated, is of the greatest utility to such as are skilled in astronomical concerns, when they discourse concerning the eclipses of the sun and moon. For, employing this theorem, they shew that three consequent eclipses, distant from each other by an equal space, cannot subsist. I say, in such a manner, that the second may be distant from the first by as great a space of time as the third from the second. For example, if the second is produced after the first, when six months and twenty days are elapsed; the third, will by no means be produced after the second, by the same, but by either a greater or less interval of time. But that this is the case may be demonstrated by the seventh theorem. And the institutor of the elements has not only exhibited the present as conferring to astronomy, but a multitude of other theorems and problems. For to what other end shall we say that the last problem of the fourth book was proposed, by which we are taught how to inscribe the side of a figure of fifteen angles in a circle, than for its relation to astronomy? For those who describe in a circle a quindecangle passing through the poles, will, by this means, obtain the distance of the poles of the equator from the poles of the zodiac. Since they are distant from each other by the side of a quindecangle. The institutor of the elements, therefore, appears by regarding astronomy, to have previously exhibited many things preparative to our advancement in that science. But when, at the same time, he saw that this seventh theorem is exhibited from the fifth, and proves the eighth without any variety, he assigned it the present place. The addition of Philo is, indeed, beautiful, but is not sufficiently adapted by its variety of cases to an elementary institution. And thus much in reply to the present question.

But if any one should doubt why he does not add so much in the eighth as in the fourth theorem, I mean, *that the triangles and the remaining angles are equal*; we must say, that because the equality of the vertical angle is demonstrated, it follows, that all are equal to all, by the fourth theorem. It was therefore alone necessary to demonstrate this by itself, but to assume all the rest as consequents. But it seems that the equality of the vertical angles causes the equality of the bases, and of the sides comprehending those angles.

For when the bases are unequal, the same angles will not remain, though the containing equal sides are supposed, but when the base becomes less, the angle is at the same time diminished, and while that increases, the angle also receives a correspondent increase. Nor while the same bases remain, but the sides become unequal, will the angle remain; but while they are diminished, it will be increased; and while they are increased, it will be diminished: for angles, and their containing sides, suffer a contrary passion. Thus, if upon the same base, you conceive the sides descending to the lower part, you will diminish the sides, but increase the angle which they comprehend, and enlarge their distance from each other. But if you conceive the sides to be elevated, and to receive an addition as they rise, you will diminish the angle which they contain: for they will coincide the longer, when their vertex is more remote from the base. We may therefore certainly affirm that the identity of the basis and equality of the sides, in a triangle, determines the equality of its angle.