The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 7

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 61–64 (1792).]

PROPOSITION VII. Theorem IV.

[142]Upon the same right line, two right lines cannot be constituted equal to two other right lines each to each, *drawn* to different points, to the same parts, and having the same extremes with the two right lines first drawn.

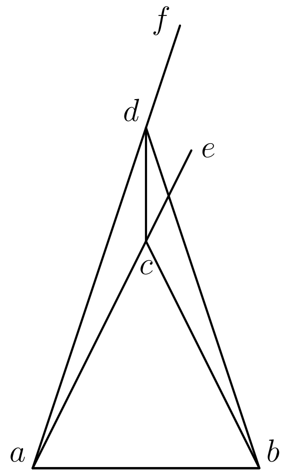
The present theorem possesses a rare property, which is not frequently found in propositions producing science. For to be formed by negation, and not by affirmation, is not their sufficiently distinguishing property. Indeed, the propositions, as well of geometrical as of arithmetical theorems, are for the most part affirmations. But the reasons of this is, (as Aristotle says) because, an affirmative universal, especially agrees with sciences, as more proper, and not indigent of negation: but a universal negative requires affirmation, in order to produce evidence; for from negatives alone, there is neither demonstration nor reasonaing. Hence, demonstrative sciences exhibit a multitude of affirmations, but rarely employ negative conclusions. However the proposition of this theorem is full of admirable diligence, and is bound with every addition, by which it is rendered so certain and indubitable, that it cannot be confuted and overturned by the efforts of opposing calumniators. For in the first place, the particle *upon the same right line*, is assumed, lest we should exhibit upon *another*, two right lines equal each to each, and employ the proposition for the purpose of circumvention. In the

[142]Mr. Simson in his note to this proposition observes, that he thought proper to change its enunciation, so as to preserve the same meaning; "because (says he) the literal translation from the Greek, is extremely harsh, and difficult to be understood by beginners." Whatever difficulty learners may find in conceiving this proposition abstractedly, is easily removed by its exposition in the figure; and therefore, I conceive, that Mr. Simson acted very injudiciously in altering its enunciation. Besides, the following comment of Proclus shews, that there is great beauty in Euclid's statement of this proposition; the greatest part of which is lost in Mr. Simson's indiscreet alteration. It would appear strange that such liberties should be taken by one, who professes in his preface, *to remove blemishes, and restore the principal books of the Elements to their original accuracy*, if Mr. Simson had not informed us in his note; that the present Commentary of Proclus unfolding the beauty and *accuracy* of this proposition, is to *trifling* to merit a relation! See more concerning this proposition in the note to Prop. 5.

second place, he does not say upon what right line, to constitute two right lines simply equal to two (for this is possible) but *each to each*. For what wonderful thing is it, that he should take both equal to both, who extends one of the constituted lines, and contracts the other? But each to each, (says he) is impossible. In the third place, he adds the particle *to different points*. For what, if some one, when he has formed two lines equal to the first two, each to each, should connect these with those in the same point, which joins the subject right lines in the vertex; and should constitute these? For the extremes of equal right lines perfectly coincide. In the fourth place, he adds the particle *to the same parts*[143]. For which if one subject right line being given, we should place two of the right lines on one side, and the other two on the opposite side, so that this common right line should be the basis of the two triangles with opposite vertexes? Lest, therefore, we should form an erroneous figure, and charge our deception on the institutor of the Elements, he ads the particle *to the same parts*. In the fifth place, he subjoins, *having the same extremes with the two right lines first drawn*. For it is possible to constitute *upon the same right line*, *two right lines equal to two*, *each to each*, *drawn to different points, and to the same parts*, by employing the whole right line, and constructing upon it, these two right lines, but then the lines last drawn, will not have the same extremes with those constituted at first. For if we conceive in a quadrangle two diagonals drawn on one of its sides, two lines shall be equal to two; a side and diameter to its parallel side, and the other diameter. But in this case the equal right lines will not have the *same* extremes. For neither the parallel sides, nor the diameters, will mutually possess the same extremes; and yet they will be equal. These distinctions, therefore, being preserved, the truth of the proposition, and the certainty of the reasoning, is evinced.

But perhaps, some, notwithstanding all these terms producing science, will dare to object, that these hypotheses being admitted, it is possible to effect what the geometrician affirms to be impossible. For let there be a right line $a\,b$, and upon this two lines $a\,d$, $d\,b$, equal to two $a\,c$, $c\,b$, and let the former be external to the latter, being drawn to different points $d\,c$, and terminated in the same extremes a and b. Let $a\,c$ too, be equal to $a\,d$: but $b\,c$ to $b\,d$. This objection, then, we shall confute, by connecting the line $d\,c$, and producing the lines $a\,c$ and $a\,d$, to the points $e\,f$. For these being constructed, it is manifest that the triangle $a\,c\,d$ is isosceles, $a\,d$, being equal to $a\,c$, from hypothesis; and the angles under the base $e\,c\,d$, $f\,d\,c$ are equal. The angle $f\,d\,c$, therefore, is greater than the angle $b\,d\,c$. Much more then is the angle $b\,c\,d$ greater than the angle $b\,d\,c$. But again, because the line $d\,b$, is equal to

[143]See the comment of Clavius on this proposition.



the line $b\,c$, the angles also at the base are equal, i. e. the angle $b\,c\,d$, to the angle $b\,d\,c$. The same angle, therefore, is both greater and equal, which is impossible. And this is what we said in our exposition of the fifth theorem, that though the equality of the angles under the base, was not useful to the demonstrations of the following theorems, yet it produced the greatest utility in the solution of objections. For in the present instance we have confuted the objection, by inferring that, because $a\,c$, and $a\,d$ are equal, the angles $e\,c\,d$, and $f\,d\,c$, are also equal. In a similar manner in other theorems, it will appear to be peculiarly useful for the solution of doubts.[144].

But if any one should say that there may be constituted upon the right line $a\,b$, right lines $b\,d$, $b\,c$, equal to the right lines $a\,c$, $a\,d$, of which $b\,c$ may be equal to $a\,c$, but $b\,d$ to $a\,d$; and that in this case they will be drawn to different points a and b, to the same parts, and will have the same extremes with $a\,c$, and $a\,d$, viz. c, and d, what shall we reply to this assertion? Shall we say that it is requisite to constitute the first lines, upon the right line $a\,b$, and their equals upon the same right line? For this is what the institutor of the Elements affirms in the proposition. But here, $a\,c$ and $a\,d$, are not constituted upon the right line $a\,b$, but only on one of its points. Hence, the lines $a\,c$, $c\,b$, and $a\,d$, $d\,b$, which stand on the right line $a\,b$, are different from the right lines, which were placed in the beginning, and to which they ought to be constituted equal. Though at the same time it is necessary that the right lines constituted upon $a\,b$, should be equal to those constituted on $a\,b$. And thus much may suffice for objections against the present question. But that the present theorem is exhibited by the institutor of the elements, by a deduction to an impossibility, and that this impossible opposes the common

[144]And from hence, also appears the emptiness and arrogance of Mr. Simson's note to this proposition, which we have already exploded.

conception, affirming that *the whole is greater than its part*; and that *the same thing cannot be both greater and equal*, is sufficiently manifest. But this theorem seems to have been assumed for the sake of the eighth theorem. For it confers to its demonstration, and is neither simply an element, nor elementary: since it does not extend its utility to a multitude. And hence, we find it very rarely employed by the geometrician.