The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 5

Transcribed by David R. Wilkins

August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 49–55 (1792).]

PROPOSITION V. Theorem II.

The angles at the base of an isosceles triangle are mutually equal; and the equal right lines being produced, the angles under the base shall be mutually equal.

Of theorems some are *simple*, but others *composite*. I call those *simple*, which, both according to hypotheses and conclusions, are indivisible, possessing one *datum*, and one object of investigation. Thus for example, if the institutor of the elements had said, *every iscosceles triangle has the angles at the base equal*, it would have been a simple theorem. But theorems are composite, which are composed from many particulars, either having composite hypotheses, or conclusions from a simple hypothesis, or both. And of these, some are *complex*, but others *incomplex*. The *incomplex* are such composites as cannot be divided into simple theorems, as the fourth proposition. For in this, both the *datum* is composite, and its consequent, yet it is impossible that the *datum* can be divided into things simple, and become theorems. For if a triangle has its sides alone equal, or the angle at the vertex, the same consequences will not ensue. But the *complex* are such as may be divided into things simple, as the theorem which says, *triangles and parallelograms of the same altitude, have the same proportion as their bases*. For it is possible to say by division, that *triangles of the same altitude, have the same proportion as their bases*, and in parallelograms after a similar manner. But of all composites, some are composed according to the conclusion, being excited from the same hypothesis: but others have their conclusion according to hypotheses, and infer the same conclusion in all: and others, lastly, are composed both according to the conclusion, and according to hypotheses. *Composition*, therefore, in the present case, is according to the *conclusion*, for there are three particulars concluded in this theorem, *that the bases are equal, that the triangles are equal, and that the remaining angles, under the base, are equal to the remaining angles*. But composition, according to *hypothesis*, is found in the common theorem *of triangles and parallelograms of the same altitude*. And *according to both*, in the theorem that *the diameters both of circles and ellipses, bisect as well the spaces as the lines containing the spaces*. But of *complex* theorems, some are universal: but others conclude that which is universal from particulars. For if we should say that a diameter divides a circle, ellipsis, and parallelograms, we receive, indeed, every part of the complex, not universally, but we make that universal which is composed from all. But if we should say, that *in a circle, all lines passing through the centre, mutually bisect each other, and make equal angles of all the segments*, we should affirm a universal. For in an ellipsis all the angles of the segments are not equal, but those only which are formed by the diameter. But these compositions are entirely fabricated, for the sake of geometrical brevity and resolutions. For many things incomposite are not resolved, but composites alone afford convenience to a resolution tending to principles.

In consequence of these previous considerations then, we must call the fifth theorem a *composite*, and a composite, both with respect to the *datum*, and the *object of investigation*; and this the institutor of the elements exhibiting, divides this theorem, being one, and gives a separate position to the *data*, and the *things to be investigated*, for he says that *the angles at the base of an isosceles triangle are equal*; and again, that *the equal sides being produced, the angles under the base are equal.* For we must not think that there are two theorems, but one; and that this is a composite, both according to the *data*, and *thing sought*: and that each of these composites is perfect and true. Hence, conversion also is true in each. For if the angles at the base are equal, the triangle is isosceles: but if those under the base are equal, the equal right lines are produced, and the triangle is isosceles. But the institutor of the elements *converts* the equality of the angles at the base; but not the equality of those under the base, though this is likewise true; the reason of which we shall shortly explain. But we shall now, in the first place, enquire on what account he demonstrates that the angles under the base are equal. For he never employs this in the construction or demonstration of other problems or theorems. It may be doubted, therefore, why, since it is useless, it was requisite to insert it in the present theorem? To this we must reply, that though it is never employed in the elements, yet it is most useful for the destruction of objections, and the solution of oppositions to theorems.[139]. But it is artificial, and belongs to science to prepare solutions of

[139]Mr. Simpson, in his note on the 7th proposition of this book, positively asserts, that it contains two cases, though there is but one in the Greek text; and ridicules Proclus for asserting that the second part of the present proposition was added, in order to solve objections which might be urged against the seventh. But that Euclid never added any more than one case, is, I think, evident, not only from no such case being found in the Greek copies so early as the age of Proclus; but from his not converting it in the 6th proposition. Besides, it is employed with advantage in the solution of objections against the 9th proposition, as the reader will perceive in its commentary; and the objection there started merits the appellation of a case, as much as the 7th. But Mr. Simpson seems to have been ignorant of Euclid's design in these elements;—the tradition of that only which is accommodated to an elementary institution. Hence, Euclid every where avoids a multiplicity of cases; and anticipates objections where he foresees they may be urged.

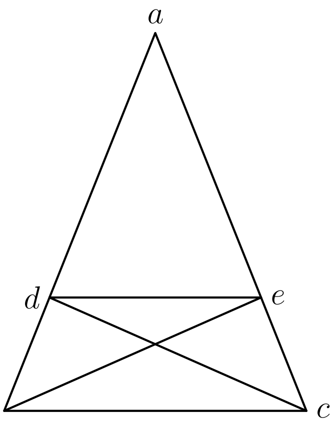
things resisting its propositions, and to provide subsidies of answers; that not only true demonstrations may be fabricated from things previously demonstrated, but that from hence confutations of error may be produced. And from this geometrical order, you will likewise receive a rhetorical emolument. For he who can effect this in the discourses of rhetoric, who can foresee the oppositions to his following heads, and previous to their delivery, can first of all prepare solutions of them to others, he, indeed, will fabricate in a wonderful manner, a most excellent mode of disputation. The institutor of the elements, therefore, teaching us this in reality, previous to the theorems by which we solve opposing objections, employing such as are now exhibited, at the same time demonstrates, that the angles under the base of an isosceles triangle, are equal, and thus prepares a confutation of the falsehood such objections contain. But that from the present theorem we may solve the objections urged in the seventh and ninth propositions, will be perspicuous as we proceed. Hence, it appears, why Euclid does not convert the latter part of his theorem in the sixth, because it does not produce a principal utility, but

---

Mr. Simpson adds in support of his dogmatical assertion, "that the translation from the Arabic has this case explicitly demonstrated." As if an Arabic translation was of greater authority than the Greek text which Proclus consulted! And lastly, he concludes, with observing, that "whoever is curious, may read what Proclus says of this in his commentary on the 5th and 7th propositions; for it is not worth while to relate his trifles at full length." If an accurate knowledge of the nature, beauty, and tendency of a science, or a collection of scientific propositions, is trifling, Proclus, indeed, deserves this accusation; as I doubt not the liberal reader, is, by this time, fully convinced. But Mr. Simpson was no philosopher; and therefore the greatest part of these Commentaries must be considered by him as trifles, from the want of a philosophical genius to comprehend their meaning, and a taste superior to that of a *mere mathematician*, to discover their beauty and elegance. It is common, indeed, to hear geometricians of the present day exclaiming, *What need of a comment on Euclid! Is he not persicuous to every one?* I will readily admit that such gentlemen know enough of geometry for all mechanical and sensible purposes: but I fear they are totally ignorant of its end; and have never dreamt that when properly studied it is the handmaid of true philosophy, the purifier of the rational soul, and the bridge by which we may pass from the obscurity and delusion of a material nature, to the splendor and reality of intellectual vision. I add farther, that I am greatly inclined to doubt, whether such geometricians ever considered what kind of subsistence geometrical forms possess? Whether they have any certainty, or are only imaginary? Where these forms, if real, reside? And a multitude of other questions which are discussed in these Commentaries. And lastly, what is most material of all, if geometry be a science, what science itself is? This last question, indeed, they would doubtless consider so *trifling* and easy of solution, that they would readily and confidently answer with young Theætetus in Plato, "that sciences are such things as may be learned from Mathematicians, geometry, and the like; shoe-making, and other mechanical arts; and that all, and each of them are no other than sciences.!" To which admirable definition we may justly reply in the words of Socrates, "Generously and magnificently O my friends, when interrogated concerning *one thing*, have you given instead of something simple, things *many* and *various*."

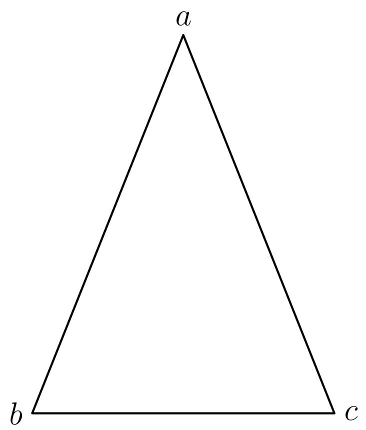
confers to our advantage, accidentally, with respect to the whole of science.

But if any one should desire us without producing the equal right lines, to prove the angles at the base of an isosceles triangle equal, (for it is not requisite to demonstrate the equality of these, by those under the base) by transposing, in a manner, the construction, and fabricating those constructions within, which are made without the isosceles triangle, we may exhibit the thing proposed. Thus let $a\,b\,c$ be an isosceles triangle, and in the side $a\,b$, take any point d, and from $a\,c$, take $a\,e$, equal to $a\,d$, and draw the lines $b\,e$, $d\,c$, $d\,e$. Because, therefore $a\,b$ is equal to $a\,c$, and $a\,d$ to $a\,e$, and the angle a



is common, $b\,e$ also shall be equal to $c\,d$, and the remaining angles to the remaining angles. Hence, the angle $a\,b\,e$, is equal to the angle $a\,c\,d$. Again, bacause $d\,b$ is equal to $e\,c$, and $b\,e$ to $d\,c$, and the angle $d\,b\,e$ to $e\,c\,d$; hence, the base, since it is common to both, is equal to itself, and all are equal to all. The angle, $e\,d\,b$, therefore, is equal to the angle $d\,e\,c$: and the angle $d\,e\,b$, is equal to the angle $e\,d\,c$. Hence, since the angle $e\,d\,b$, is equal to the angle $d\,e\,c$, from which the equal angles $d\,e\,b$, $e\,d\,c$, are taken, the remaining angles $b\,d\,c$, $c\,e\,b$ are equal. But the sides also $b\,d$, $d\,c$, are equal to the sides $c\,e$, $e\,b$, each to each, and the base $b\,c$ is common. All, therefore, are equal to all. Hence the remaining angles also, subtending equal sides, are equal. The angle, therefore, $d\,b\,c$, is equal to the angle $e\,c\,b$. For the angle $d\,b\,c$, subtends the line $d\,c$: but the angle $e\,c\,b$, the line $e\,b$. The angles, therefore, at the base of an isosceles triangle, are equal, the equal right lines not being produced.

But Pappus demonstrates this yet shorter, without any addition in the following manner. Let $a\,b\,c$ be an isosceles triangle, having $a\,b$ equal to $a\,c$. We must conceive, therefore, this one triangle as if it was two, and reason thus. Because $a\,b$ is equal to $a\,c$, and $a\,c$ to $a\,b$, the two sides $a\,b$, $a\,c$, are equal to the two $a\,c$, $a\,b$, and the angle $b\,a\,c$, is equal to the angle $c\,a\,b$, (for it is the same.) All, therefore, are equal to all. The base $b\,c$, to the base $c\,b$. But the



triangle $a\,b\,c$, to the triangle $a\,c\,b$; and the angle $a\,b\,c$, to the angle $a\,c\,b$, and the angle $a\,c\,b$, to the angle $a\,b\,c$. For they subtend equal sides, i.e., $a\,b$, $a\,c$. The angles, therefore, at the base of an isosceles triangle, are equal. And it seems that Pappus invented this mode of demonstration, when he considered that the institutor of the elements also, in the fourth theorem, when he had united two triangles, and had made them mutually coincide, thus forming one of two, by this means observed their equality throughout. In like manner it is possible, that we also, by an assumption contemplating two triangles in one, may demonstrate the equality of the angles at the base. Thanks, therefore, at to be given to the ancient Thales for the invention of this theorem, as well as a multitude of others. For he, first, is said to have perceived and affirmed, that the angles at the base of every isosceles triangle are equal: and after the manner of the ancients, to have called them similar. But still more deserving of praise are those moderns, who have yet more universally demonstrated (among which number is Geminus) that equal right lines falling from one point, on a line of similar parts, form equal angles. For Geminus using this theorem shews, that there are only three lines, and not more of similar parts, the *right*, the *circular*, and the *cylindric helix*; and this is properly universal, to which this symptom first agrees, just as the possession of two sides greater than the third, is shewn to be essentially inherent in every triangle. It is not, therefore, the property universally of every isosceles, though it belongs to every one, to possess angles at the base equal: but of equal right lines falling on a line of similar parts. For to subtend equal angles, is in these primarily inherent.