The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 3

Transcribed by David R. Wilkins

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[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 2, pp. 37–41 (1792).]

PROPOSITION III. PROBLEM III.

Two unequal right lines being given, from the greater to cut off a part equal to the less.

The third problem, likewise, has a variety of cases. For the given unequal right lines are either mutually distant from each other, as with the institutor of the elements, or they are united according to one extreme; or the one cuts the other according to one of its extremities, and this in a two fold manner. For either the greater cuts the less, or the less the greater. But if they are united according to one extreme, the demonstration is manifest. For employing the common extremity as a centre, and the less of the lines for an interval, you will describe a circle, and cut off from the greater, a part equal to the less; since as much as the circle intercepts within itself, will be equal to the less. But if the one cuts the other according to its extreme, either the greater will cut the greater, or the contrary. And if they mutually cut each other, they will either be mutually cut into equal parts, or into unequal; or the one will be cut into equal, and the other into unequal parts, and this in a two-fold respect. For all these present us with an admirable variety of exercise, some of which, out of a many, we shall exhibit. Let there be given



the unequal right lines ab, cd, the greater of which is cd, and let it cut abin one of extremities c; then with the centre a, but interval ab, let a circle bf be described, and let an equilateral triangle a e c be constructed upon ac, and produce ea, ec. Again, with centre e, but interval ef, let the circle gfbe described; and with the centre c, and interval cg, the circle gl. Because therefore ef is equal to eg (for the centre is e) of which ea is equal to ec, the remainder af, shall be equal to the remainder cg. But af is likewise equal to ab; for the centre is a. Hence, cg will be equal to ab, and this is equal to cl, for the centre is the point c: ab, therefore, is equal to cl, which was required to be done.

But let c d be less than a b, and let it cut a b according to its extremity c; either, therefore, it will cut it in the middle, or not in the middle. Let it in the first place cut it in the middle; c d, therefore, is either the half of a b, and a c is equal to c d, or it is less than half. And in this case with the centre c,



and interval cd, describe a circle, and you will cut off from ab a part equal to cd: Or it is greater than half; and then at the point a, placing af, equal to cd, and describing a circle with the centre a, and interval af, you will cut off from ab a part equal to af, that is to cd. But if cd does not cut



a b in the middle, c d shall either be its half, or greater than the half, or less. If therefore c d is the half, or less than the half of a b, employing c as a

centre, and cd as an interval, you will cut off from ab, a part equal to cd, as was required to be done. But if cd is greater than the half, again at the



point a^{134} placing a f equal to cd, you will accomplish the same. For with the centre a, but interval a f, you will describe a circle, cutting off from a ba line equal to a f, that is, to cd. But if they mutually intersect, as cd, ab, then with the centre b, but interval ba, describe the circle a f, and let bcbe extended to the point f. Because, therefore b f, cd are the two unequal right lines, and cd cuts bf, according to its extremity, it is possible from cd to make a line equal to bf; for this has been shewn in the first case of this problem. It is therefore possible, that a line equal to a, b may be cut



off from c, d; for a b and b f are mutually equal. Having, therefore, received these cases from division, we have endeavoured to exhibit their variety. But the demonstration of the elementary institutor is admirable, since it accords with all the preceding constructions. And it is possible, in every position, at the extremity of the greater, to place a line equal to the less, and using the same extreme as centre, and placing the interval to describe a circle, which

¹³⁴See the third figure of this problem.

shall cut off from the greater, a line equal to the less, whether they mutually intersect, or one cuts the other, or they are constituted in a still different position.