The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 2

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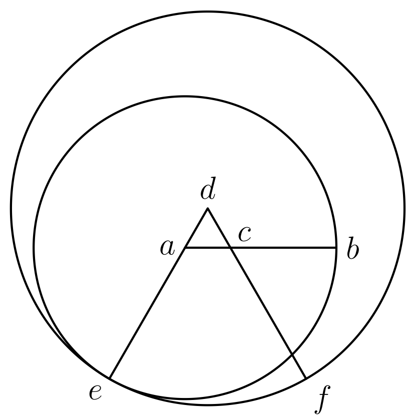
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 33–37 (1792).]

PROPOSITION II. Problem II.

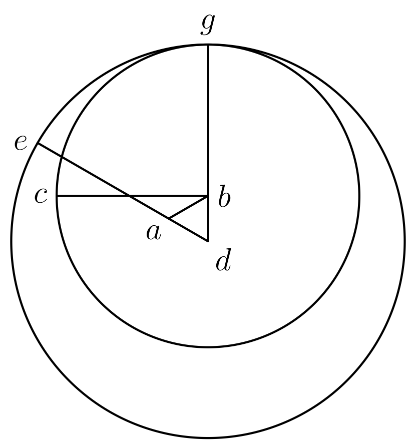
To a given point to place a right line equal to a given right line.

Of problems, as well as of theorems, some are without case, but others possess a multitude of cases. Whatever, therefore, have the same power acceding to many descriptions, and when their positions are changed, preserve the same mode of demonstration, these are said to have *case*; but such as proceed according to one position only, and one construction, are without *case*; for simply, *case*, appears about the construction both of theorems and problems. The second problem, therefore, has many cases; but a point is given in it *in position*, since it can only be given in this manner; but a right line, both in form and position, (for it is not simply *line*, but of such a kind.) For it is here enquired, *how to a given point to place a right line equal to a given right line*. But it is manifest that the point is entirely in the subject plane, in which the right line exists, and not in one more elevated. For in all problems and theorems respecting planes, we must conceive that one plane is subjected. But if any one should doubt how a line is to be placed equal to a given right line, for what if the given line be infinite? Since the present datum pertains both to finite and infinite: for every datum signifies that which is proposed and supposed by us for the sake of investigation. But this Euclid himself declares, sometimes, saying, *upon a terminated right line to construct an equilateral triangle*; but at other times, *upon a given infinite right line to let fall a perpendicular*. In answer then to this doubt, we must say, that when he orders us to place the line equal to a given right line, at a given point, he sufficiently evinces that the given line is finite; for every thing placed at a point, is terminated according to that point. Hence, the line equal to that which is given, must have a much prior termination. At the same time, therefore, in which he says, *to a given point*, he terminates both the given right line, and its equal which is investigated.

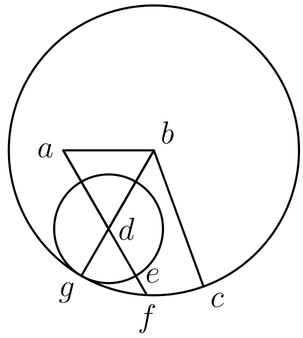
But that the cases of the present problem are formed from the various position of a point, is manifest. For the given point is either placed external to, or in the given right line; and if in it, it will either be one of its extremities, or it will be situated within the extremes; and if external, it will either have a lateral position, so that a line drawn from it to the extremity of the given line will form an angle, or a direct position; so that if the line were produced, it would coincide with the external point. But the geometrician, indeed, considers the point as external, and receives it according to a lateral position; however, for the sake of exercise, all the positions are to be assumed, the more difficult of which we shall exhibit. For let there be given a right line $a\,b$, and a given point c, which lies between its extremes, and let there be constituted according to the doctrine of the elements, an equilateral triangle upon the right line $a\,c$, and let $d\,c$, $d\,a$ be produced; then, with the centre a, and the interval $a\,b$, let the circle $b\,e$ be described. And again, with the centre d, but with the interval $d\,e$, let the circle $d\,f$ be designed. Because, therefore, a is



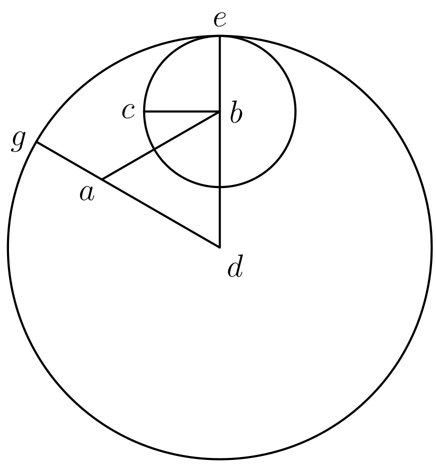
the centre, $b\,a$ is equal to $a\,e$; and hence, $d\,e$ is equal to $d\,f$, the parts of which, $d\,a$, $d\,c$, are equal: for the triangle $d\,a\,c$ was established as equilateral. The remainder, therefore, $a\,e$, is equal to $c\,f$; but $a\,e$, as it was shewn, is equal to $a\,b$, and hence $c\,f$ is equal to $a\,b$. To a given point, therefore, c, a right line $c\,f$ is placed equal to $a\,b$. With respect to the position of the point then, so many cases arise. But there are many more with respect to the constitution



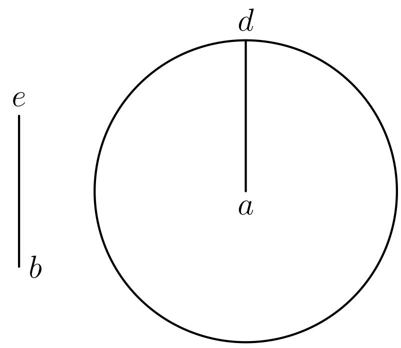
of the equilateral triangle, the extension of its sides, and the description of circles. For let there be assumed, as in this element, a point a, and a right line $b\,c$, but let $b\,a$ be extended. The equilateral triangle, therefore will not be constituted on $b\,a$, with its vertex above (because there is no place for it), but beneath; let it therefore be $a\,d\,b$; $a\,d$, therefore, is either equal to $b\,c$, or greater or less. If then it be equal that which was required is performed. But if less with the centre b, and the interval $b\,c$, let a circle be described, and let $a\,d$, $d\,b$ be produced to the points e and g; and with the centre d, but the interval $d\,g$, let a circle $g\,a$ be designed. Because, therefore, $d\,g$ is



equal to $d\,e$, for they are drawn from the centre; and likewise because $a\,d$ is equal to $d\,b$, for the triangle is equilateral, the remainder $a\,e$ is equal to the remainder $b\,g$. But $b\,g$ is also equal to $b\,c$, for they proceed from the centre; and hence, $a\,e$ is equal to $b\,c$, which was required to be done. But if $a\,d$ is greater than $b\,c$ (for this is the last case), then with the centre b, and the interval $b\,c$, let a circle $e\,c$ be described. The line $d\,b$, therefore, shall cut the circle $e\,c$. Again, with the centre d, and interval $d\,e$, let the circle $e\,g$ be described. Because therefore, d is the centre of the circle $g\,e$, $g\,d$, is equal to



$d\,e$. But $d\,a$ was also equal to $d\,b$; the remainder, therefore, $a\,g$ is equal to the remainder $b\,e$. But $b\,e$ is equal to $b\,c$, for both proceed from the centre. Hence, $a\,g$ is equal to $b\,c$; and it is placed at the point a, as was required to be done. And though there are many other cases, the description of the above is sufficient for our present purpose. For from these it is possible for the more curious to exercise themselves in the rest. But formerly some destroying the construction and variety of this problem, reasoned thus. Let a be a given point, but $b\,e$ a given right line, and with the centre a, but with an interval equal to $b\,e$, let a circle $d\,e$ be described. Then let a certain right line $a\,d$ be extended from the point a to the circumference; and this shall be equal to $b\,e$: for the magnitude of the line from the centre, was equal to that of $b\,e$: and so that is done which was required. But he who thus reasons, *begs*, in the



very beginning. For when he says with the centre a, but interval $b\,e$ describe a circle $e\,d$, he receives, in a certain manner, a line equal to $b\,e$, placed at the extremity a; and preserving the Petition, he makes one extremity of the interval a centre; but with the other describes a circle: however, in this case, the centre is in one place, but the interval in another. We by no means, therefore, approve this method of demonstration.