The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 1

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 18–33 (1792).]

PROPOSITION I. Problem I.

Upon a given terminated right line to describe an equilateral triangle.

Since all science is two-fold, and one is conversant about immediate propositions, but another about things, which are exhibited and provided from the propositions, and universally about the consequents to principles; this, again, divides itself in geometrical discourses, into the solution of problems, and the invention of theorems. And problems, indeed, geometry denominates things in which it proposes to procure, manifest, and fabricate that, which, in a certain respect, has no existence; but it calls theorems, things in which it appoints to perceive, know and demonstrate that which either exists, or does not exist. For problems command us to undertake the origin, positions, applications, descriptions, inscriptions, circumscriptions, coaptations, and contacts of figures, and every thing of this kind: but theorems endeavour to procure our assent to symptoms, and things essentially inherent in the subjects of geometry, and to convince by demonstrations. For geometry discourses concerning every object of enquiry, which is possible to be effected, referring some things to problems, but others to theorems; since it enquires concerning the *what*, in a two-fold respect: for it either seeks for the reason and intelligence of the thing; or for intelligence, and the essence of the subject. I say, for example, as when it requires what a line of similar parts may be: for in an enquiry of this kind, it either desires to find the definition of such a line, as, *that a line of similar parts is that which has all its parts agreeing with all*; or to receive the species of lines of similar parts, as that it is either *right*, or *circular*, or a *cylindric helix*. Besides, prior to this, in enquires, by itself, concerning the *is*, in this especially in its determinations, agitating, whether the object of its enquiry is possible or impossible, what place it possesses, and in how many ways. It likewise seeks concerning the *what kind*; for when it considers the essential accidents of a triangle, circle, and parallels, it is manifest, that in such cases it seeks after the *what kind*; but many have thought that geometry very little contemplated the *cause*, and *the why*. And of this opinion is Amphinomus, led by the decisions of Aristotle: but (says Geminus) an enquiry into these may be found in geometry. For does it not belong to geometry to enquire for *what cause* infinite equilateral multangles may be inscribed in circles; but to describe solid equilateral and equiangular multangles, and constructed from similar planes, in spheres, is impossible? To whom does an investigation of this kind belong, except to a geometrician? When, therefore, to geometricians the syllogism is by an impossibility, they alone desire to find the symptom; but when by a principal demonstration, then again if the demonstrations are in that which is particular or partial, the cause is not yet manifest; but if in that which is universal, and in all similars, the *why* becomes immediately manifest: and thus much concerning objects of enquiry.

But every problem and theorem which receives its completion from its own perfect parts, ought to possess in itself all the following parts: *proposition*, *exposition*, *determination*, *construction*, *demonstration*, and *conclusion*. But of these, *proposition*, informs us what the object of enquiry is from a given datum; for a perfect proposition is composed from both; but *exposition* receiving the datum essentially, prepares for the question. Again, *determination* separately explains the things sought for according to the *what*; but *construction* adds to the datum what is wanting to the investigation of the thing sought; and *demonstration* skilfully collects the proposition from the concessions. But the *epilogue*, or conclusion, is again converted to the proposition, by confirming that which is exhibited. And so many, indeed, are all the parts of problems and theorems; but *proposition*, *demonstration*, and *conclusion*, are especially necessary, and exist in all; for it is requisite that the thing sought for should be previously known, and this this should be shewn by proper mediums, and that what is exhibited should be concluded; and it is not possible that any one of these three can be wanting; but the rest are, indeed, received in many places; but in many, because they produce no utility, are omitted. For *determination* and *exposition* are not found in the problem, which says, *to construct an isosceles triangle, which will have each of the angles at the base double of the other*; but *construction* has frequently no subsistence in many theorems, the demonstration being sufficient to exhibit the thing proposed from the data, without any addition. When, therefore, shall we say that *exposition* fails, when no datum is given in a proposition? Because, though *proposition*, for the most part, is divided into *datum*, and *the thing sought for*, yet this is not always the case; but sometimes *the thing sought for*, alone affirms that which it is requisite to know or effect, as in the aforesaid problem; for it does not previously say from what datum it is requisite to construct an isosceles triangle, which shall have each of the angles at the base, double of the remaining one; but that it is required to effect this. And here, indeed, the admission of the proposition takes place from things previously known; for we must know the meaning of the terms *isosceles*, *equal* and *double* (since this, as Aristotle observes, is the property of all ratiocinative discipline[131]), yet nothing is subjected to us as in other problems, as in that which says, *to bisect a given terminated right line*. For here the right line is given, but we are ordered to divide it into two parts; and the datum is separately determined from the object of enquiry. When, therefore, a proposition has both of these, then also *determination* and *exposition* are found; but when the datum is deficient, these also fail, since *exposition* and *determination* belong to the datum: for this will be the same with the proposition. Indeed, what else do we say, when determining in the aforesaid problem, unless that it is requisite to find an isosceles of this kind? But such was the proposition: if then the proposition has neither this *datum*, or *thing sought*, *exposition* will, indeed, be silent, because there is no datum; but *determination* will be neglected, lest it should become the same with the *proposition*: but you may find many other problems of this kind, especially in arithmetic, and in the tenth book of these Elements, as, *to find a medium comprehending two right lines commensurable in power, and every thing of this kind*.

But every datum may be given in these four modes, either in *position*, or *proportion*, in *magnitude* or *form*; for a point, indeed, is given in *position* only, but a line and the rest in all the four. Thus, when we say, *to bisect a given rectilineal angle*, we declare the species of the angle given, as that it is right lined, lest we should also seek to bisect a curvilinear angle by the same methods. But when we say, *from the greater of two unequal right lines, to cut off a part equal to the less*, the lines are given in magnitude; for the less and the more, finite and infinite, are the proper predications of magnitude. But when we say, that *if four magnitudes are proportional, they shall be also alternately proportional*, the same proportion is given in the four magnitudes: but when it is requisite, *from a given point to place a right line equal to a given line*, then the point is given in position. From whence, since position may be various, construction also receives variety; for the point is given either without the right line, or in the right line, and in the extremity, or without the extremity of the right line. Since, therefore, a datum has a four-fold acceptation, it is manifest, that exposition also is four-fold; but sometimes it connects two or three modes. Again, we find that demonstration sometimes possesses things proper to demonstration, exhibiting the thing sought for from mediate definitions; for this is the perfection of demonstration, but that sometimes it argues from certain signs. And it ought not to be concealed, that geometrical discourses have every where that which is necessary, on acount of the subject matter, but are not every where perfected by demonstrative methods. For when, because *the external angle of a triangle is equal to the*

[131]See Section second, of the Dissertation, in Vol. I. of this work.

two internal and opposite ones, it is shewn, that *the three internal angles of the triangle are equal to two right*, how is this demonstration from the cause? And is not a sign the medium in this case? For the external angle not yet existing, since the internal angles exist, they are equal to two right, since it is a triangle, though the side is not produced; but when, by a description of circles, the triangle, which is constituted, is shewn to be equilateral, the apprehension takes place from the cause. For we say, that the similitude and equality of the circles is the cause of the triangle's equality with respect to its sides.

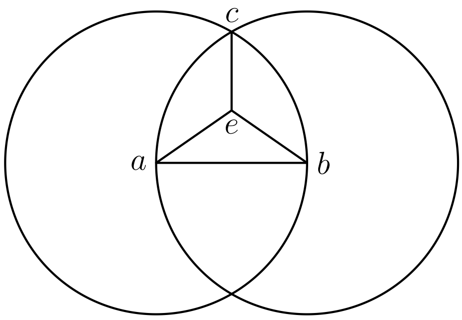
But geometrical discourses are likewise accustomed to make the conclusion, in a certain respect, two-fold. And this, when they exhibit things agreeable to the data, and reason universally, recurring from a particular conclusion to that which is universal; for when they do not use the property of the subjects, but placing the data before our eyes, describe an angle or right line, they think that which is concluded in this, is to be concluded in every thing similar: they pass on therefore to *universal*, lest we should think that the conclusion is particular. But their transition is effected in the best manner, since they employ, in demonstration, *the things placed*, not considered as such, but considered as similar to others: for it is not because such a particular angle is proposed that they effect a bipartite section, but because it is rectilineal only. But quantity, is indeed, proper to the proposed angle; but rectilineal is common to all right lines: let then the given angle be a right one. If therefore, we receive rectitude in the demonstration, we cannot pass to every species of right lines; but if we do not subjoin its rectitude, or being right angled, but alone consider its being rectilineal, the discourse may be adapted to all right lined angles; and all that we have previously observed we may contemplate in this first problem. For that it is a problem, is evident, since it commands us to construct an equilateral triangle: but *proposition* in this, consists from a *datum* and *thing sought.* For a terminated right lines is given, but it is *enquired* how an equilateral triangle may be constructed upon it, and the datum indeed precedes, but the thing sought follows; so that we may say, by conjoining the two, *if there be a terminated right line, it is possible to construct upon it an equilateral triangle*; for a triangle cannot be constructed without the existence of a right line, since it is comprehended by right lines; nor upon an unlimited line, for an angle cannot be constructed unless it is made on one point, but in an infinite line there can be no extremity or bounding point. But after proposition, *exposition* follows, as, *let there be given a terminated right line.* And here we may see that *exposition* alone produces the *datum*, but by no means subjoins *the thing sought*; but after this we shall find *determination*: *it is required upon the given terminated right line to construct an equilateral triangle*; and here we may observe that *determination*, is in a certain respect, the cause of attention, for it makes us more attentive to the demonstration, by pronouncing the thing sought, as *exposition* causes us to be more docile, by placing the datum before our eyes. Again, after determination, *construction* follows, *from one extremity of the right line, as a centre, but with the remaininder as an interval, let a circle be described*. And again, *with the other extremity, as a centre, and with the same interval, let a circle be described; and from the common point of the sections of the circles, to the extremities of the right line, let right lines be continued*. And here we may observe, that Petitions are used in the construction, this, for one, *from every point to every point, to draw a right line*; and also this, *with every centre and interval to describe a circle*; for universally Petitions are the sources of utility to *constructions*, but Axioms to *demonstrations*; *demonstration* therefore follows, *because, then each extremity of the given right line is the centre of the circle surrounding it, the right line which reaches to the common section is equal to the given right line; hence, because the other extremity of the right line is the centre of the containing circle, the right line reaching to the common section of the circles, is also equal to the given line*. And the admonition of these, is derived from the definition of the circle, which says, that *all lines from the centre to the circumference are equal*. Each of these lines, therefore, is equal to the same; but *things equal to the same, are equal among themselves*, by the first axiom. The three right lines, therefore are mutually equal; hence, *upon this given right line an equilateral triangle is constructed*; and this, indeed, is the first *conclusion* which follows the exposition. But after this, that universal one, *upon a given right line, therefore an equilateral triangle is constructed*: for whether you make the line double of the one now proposed, or triple, or receive any one greater or less, the same constructions and demonstrations will accord. But to these he adds the particle *which was required to be done*, shewing from hence, that the conclusion is problematical; for in theorems, he adds the particle *which was required to be shewn*; the former announcing the production of something, but this the ostention and invention of a thing required. He therefore subjoins this to the conclusions, for the purpose of shewing that every part of the proposition is accomplished by this means, uniting the end with the beginning, and imitating intellect convolved, and again returning to its principle. But he does not always add the same, but sometimes the particle *which was required to be done*, and sometimes the particle *which was required to be shewn*, on account of the difference between problems and theorems: and thus, in this one problem, we have exercised and made perspicuous all this variety of considerations. But the reader ought to make a similar enquiry in the rest; investigating what propositions receive these leading properties, and in what they are omitted. Likewise in how many ways a *datum* is given, and from what principles we receive either constructions or demonstrations; for a perspicacious contemplation of these affords no small exercise and meditation of geometrical discourses.

But here it is necessary that we should briefly determine the nature of *assumption*, *case*, *corollary*, *instance*, (ἔνστασις[132]) and *induction*. They say therefore that *assumption* is often predicated of every proposition assumed in the construction of another proposition, affirming at the same time that the demonstration of such a proposition is composed of so many *assumptions*. But *assumption*, properly considered by those who are conversant in geometry, is a proposition indigent of credibility; for when either in construction or demonstration we assume any thing which has not been exhibited, but requires a reason for its admission, then that which is assumed, as of itself ambiguous, being considered as worthy of enquiry, we call an *assumption*; and this differs from Petition and Axiom, because it is demonstrable, but they are assumed without demonstration, for the purpose of giving credibility to others. But the best aid in the invention of *assumptions*, is an aptitude of cogitation; for we may see many naturally acute in solutions, and discovering them without any method, as was the case with our Cratistus, who was adapted to the investigation of a thing sought from the first and shortest methods possible; and had a natural promptitude for invention; but there are nevertheless certain most excellent methods delivered, one which reduces the thing sought, by resolution to its explored principle, which, as they say, Plato delivered to Leodamas, and from which he is reported to have been the inventor of many things in geometry: but the second is that which has a power of division; because it distributes the proposed genus into articles, but affords an occasion of demonstration, by an ablation of other things from the proposed construction. And this likewise is praised by Plato, as that which affords assistance to all sciences; but the third is that which by a deduction to an impossibility, does not of itself shew the thing sought, but confutes its opposite, and discovers the truth by accident; and thus far is the contemplation of *assumption* extended. But *case* enunciates different methods of construction, and the mutation of position, points, or lines, superficies, or solids being transposed; and in fine, all its variety is beheld about description; hence, it is also called case, because it is the transposition of construction. Again, *Corollary* is affirmed, indeed, of certain problems, as the Corollaries which are ascribed to Euclid; but Corollary is properly predicated, when, from the things demonstrated, a certain unexpected theorem appears, which

[132][DRW—The Greek spelling has been verified from Proclus's *Commentary*, edited by Friedlein, p. 210, 27. From the discussion that follows, it would appear that these 'instances' represent actual or pretended counter-examples.]

in this account they have denominated Corollary, as a certain gain, exceeding the intention of demonstrative science; but *instance* impedes the whole passage of the discourse, either opposing the construction or the demonstration: and here it is not necessary, that as he who proposes a case, ought to shew the proposition true; so he who proposes an *instance*: but it is requisite to destroy the *instance*, and convict its employer of falsehood. Lastly, *induction* is a transition from one problem or theorem to another, which being known or compared, the thing proposed is also perspicuous. For example: when the duplication of the cube is investigated, geometricians transfer the question into another to which this is consequent, i. e. the invention of two mean proportionals, and afterwards they enquire how between two given right lines, two means may be found. But Hippocrates Chius is reported to have been the first inventor of geometrical induction; who also made a quadrangle equal to a lunula, and invented many other things in geometry, and excelled all in his ingenuity respecting appellations: and thus much for these.

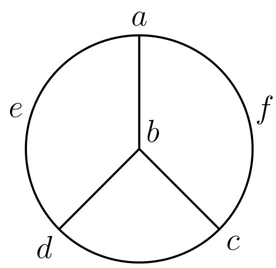
But let us return to the proposed problem: that an equilateral triangle, therefore, is the best among triangles, and is particularly allied to a circle, having all lines from the centre to the circumference equal, and one simple line for its external bound, is manifest to every one; but the partial comprehension of two circles in this problem, seems to exhibit in images how things which depart from principles, receive from them perfection, identity, and equality. For after this manner, things moving in a right line, roll round in a circle, on account of continual generation; and souls themselves, since they are indued with transitive intellections, resemble by restitutions and circumvolutions, the stable energy of intellect. The zoogonic or vivific fountain of souls too, is said to be contained by two intellects. If, therefore, a circle is an image of the essence of intellect, but a triangle of the first soul, on account of the equality and similitude of angles and sides; this is very properly exhibited by circles, since an equilateral triangle is included in their comprehension. But if also every soul proceeds from intellect, and to this finally returns and participates intellect in a two-fold respect; on this account also it will be proper that a triangle, since it is the symbol of the triple essence of souls, should receive its origin comprehended by two circles. But speculations of this kind, as from bright images in the mirror of phantasy, recall into our memory the nature of things. And here, because some object to the constitution of an equilateral triangle, thinking by this means to overthrow the whole of geometry, let us briefly answer and confute them. Zeno then, whom we have mentioned before, says, that if any one admits the principles of geometry, yet he will not obtain from common consent, things consequent to the principles, while this is not admitted, that there are not the same segments of two right lines: for unless this is given an equilateral triangle cannot be constructed. For let there be (says he) a right line $a\,b$, upon which an equilateral triangle is to be constructed. But let circles be described, and from their common section



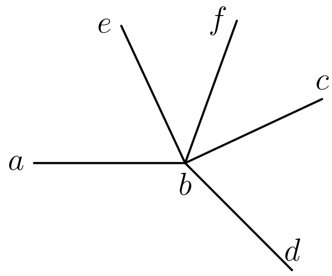
let the right lines $c\,e\,a$, $c\,e\,b$, be extended, having the common segment $c\,e$. It will therefore happen, that the lines extended from the common section, will be equal to the given line $a\,b$, and yet the sides of the triangle will not be also equal, but two will be less than the remainder, that is, than $a\,b$. And so this not being constituted, neither can the rest be constructed. Can then (says Zeno) the rest follow, though the principles are given, unless this also is previously received, that there are no common segments either of circles or of right lines? Against this objection then, we must affirm in the first place, that it was in a certain respect previously understood, that two right lines have no common segment. For the definition of a right line comprehends this property, since *that is a right line which is equally situated between its bounding points*[133]; and the equality of the interval between the points to the right line, causes that which joins the points to be one, and the shortest line; so that if any one adapts it to another line, according to one of its parts, it must also agree with the line according to its remaining part; for since it is constituted in its extremities, because it is the shortest line, it is necessary that the whole should fall on the whole. But again, this was manifestly received in the Petitions: for the Petition which says, *that a terminated right line may be produced straight forwards*, perspicuously shews that the produced line ought to be one, and produced by one motion; but if any one is desirous to receive a demonstration of this assumption, let, if possible, $a\,b$ be the common segment of $a\,c$ and $a\,d$, and with the centre b, and interval $b\,d$, let the circle $a\,c\,d$ be described; because therefore the right

[133][DRW–Proclus here quotes the Euclidean definition of a straight line. The Greek text, in Friedlein's edition of Proclus's *Commentaries* (Friedlein, p. 215, 16, 17) reads as follows: εἴπερ εὐθεῖά ἐστιν ἡ ἐξ ἴσου κειμένη τοῖς ἐφ' ἑαυτῆς σημείοις. This definition of a straight line is translated into English, by Heath and others, as requiring that the line "lies evenly with the points on itself". Thus the Euclidean definition of a straight line, referenced by Proclus here, does not specify that the points on the line are "bounding points".]

line $a\,b\,c$, is drawn through the centre, $a\,f\,c$ is a semicircle: and because the right line $a\,b\,d$ likewise is drawn through the centre, $a\,e\,d$ is a semicircle. The semicircles, therefore $a\,f\,c$, $a\,e\,d$, are equal to each other, which is impossible. But against this demonstration Zeno will perhaps say, that it is likewise

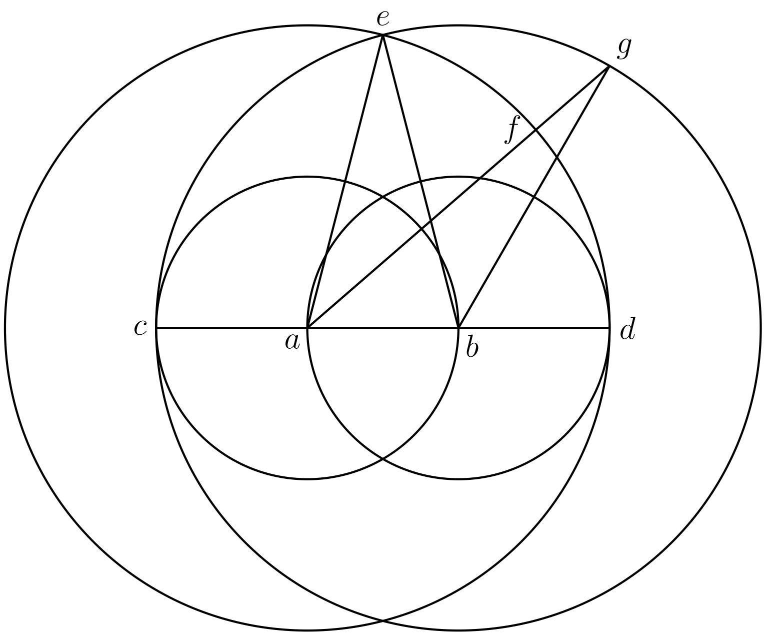


requisite to demonstrate that the diameter bisects the circle, because we previously assume that there is not a common segment of two circumferences. Thus too we take for granted, that one circumference coincides with another, or if it does not coincide, that it either falls externally or internally. But nothing hinders (he will say) that the whole may not coincide with the whole, but according to some part. But to this Possidonius rightly answers, who laughs at the acute Epicurean, as if conscious that though the circumferences do not coincide according to a part, yet the demonstration will succeed; for according to that part in which they do not coincide, the one will fall within, and the other without, and the same absurdities will follow when right lines are extended from the centre to the external circumference; for those from the centre will be equal, as well the greater which is drawn to the external, as the less which is extended to the internal circle: either therefore the whole will coincide with the whole, and they will be equal; or coinciding according to a part, it will alternately vary according to the remainder, or no part will coincide with no part; and in this case it either falls within or without: but of this, enough. But Zeno also condemns the following demonstration of this particular: Let $a\,b$ be the common segment of two right lines $a\,c$, $a\,d$, and let $b\,e$ be erected at right angles to $a\,c$, the angle $e\,b\,c$, therefore, is a right one. Hence if the angle $e\,b\,d$ is also right, they shall be equal, which is impossible; but if not, let $b\,f$ be erected a right angles to $a\,d$. The angle $f\,b\,a$, therefore, is right; but the angle $e\,b\,a$ was also right; and they are therefore mutually equal, which is impossible. This is the demonstration which Zeno opposes, assuming that which is to be exhibited afterwards; I mean from a given point to raise a right line, at right angles, to a given right line. However, Possidonius observes, that indeed, a demonstration of this kind is never to be introduced into elementary institutions; but that Zeno calumniates Geometricians using their own as a flagitious demonstration; though there is some reason in their



conduct. For there are right lines existing at right angles; since any two right lines are capable of forming a right angle; and this is previously assumed in our definition of a right angle. For we alone constitute a right angle from such an inclination; and it may perhaps be this which we have erected. Indeed, Epicurus himself, and all other philosophers admit, that not only many things possible may be supposed, but likewise many of an impossible matter, for the purpose of contemplating something consequent; and thus much concerning an equilateral triangle.

But it is requisite to construct other triangles, and in the first place an isosceles. Let $a\,b$, therefore, be a right line, upon which it is requisite to construct an isosceles triangle. Describe circles as in the construction of an equilateral triangle, and produce the line $a\,b$ on each side to the points $c\,d$; the line $c\,b$, therefore, is equal to $a\,d$. Again, with the centre b, and interval $c\,b$, let the circle $c\,e$ be described; and with the centre a, and the interval $d\,a$, the circle $d\,e$; and from the point e, in which the circles intersect each other, to the points a and b, let the lines $e\,a$, $e\,b$ be extended. Because therefore, $e\,a$ is equal to $a\,d$; but $e\,b$ to $b\,c$, and $a\,d$ is equal to $b\,c$, $e\,a$ will also be equal to $e\,b$; but they are also greater than $a\,b$. The triangle $a\,b\,e$, therefore, is isosceles, which it was required to constitute. But let it be ordered to construct a scalene triangle upon the given right line $a\,b$. Describe circles with centres and intervals, as before, and let there be taken in the circumference of the circle, whose centre is a, the point f, and let the right line $a\,f$ be extended and produced to the point g; and likewise let the right line $g\,b$ be extended. Because, therefore, a is a centre, $a\,f$ is equal to $a\,d$; and hence, $a\,g$ is greater than $a\,d$, that is, than $g\,b$. But b also is a centre, $g\,b$, therefore, is equal to $c\,b$; and hence $g\,b$ is greater than $b\,a$: but $g\,a$ is greater than $g\,b$; the three lines $g\,b$, $b\,a$, $a\,g$, are unequal; and hence, the triangle $a\,b\,g$ is scalene. Hence too, three triangles are constructed; but these things are commonly known: however, this is beautiful in these triangles, that the equilateral existing on all sides equal, is constructed by one mode alone; but the isosceles, endued with equality on two sides only, has a two-fold construction: for the given right line is either less than both the equal ones (according to



our present construction), or it is greater than both; but the scalene being unequal in all its sides, receives a triple construction; for the given right line is either the greatest of the three, or the least, or greater than the one, and less than the other; and indeed, it is proper to be exercised in each supposition, either by enlarging or contracting; but to us, what is already delivered, is sufficient. Let us now contemplate problems universally, some of which are produced simply, but others manifoldly, and others according to infinite modes. But (as Amphinomus observes) those which are simply constructed are *ordinate*: but those which receive a manifold composition, and are constructed according to number, are *middle*; and those which are varied in infinite ways, are *inordinate*. The manner, therefore, in which problems are constructed, simply or manifoldly, becomes manifest in the preceding triangles; for the equilateral is constituted simply; but of the other two, the one receives a two-fold, and the other a triple construction. But problems of the following kind, may take place in infinite modes; I mean *to divide a given right line in three proportional parts*; for if it be divided in a duple ratio, and the deficient quadrangular form, resulting from the less, be applied to the greater, it will be divided into three equal parts; but if the greater segment be more than double of the less, as for instance, triple, and a deficient quadrangular form, equal to that which results from the less, be applied to the greater, the line will be divided into three unequal parts. Because, therefore it may be divided into two parts, in infinite ways, the greater of which is either double or triple, (for multiplex proportion proceeds in infinitum), hence, it may be divided into three parts, according to infinite variations.

But it is requisite to know, that problem also is manifoldly predicated; for whatever is proposed may be called a problem, whether it is proposed for the sake of learning or operating. But in mathematical disciplines, that is properly called a problem, which is proposed for the purpose of contemplative energy. Since that which is performed in these, has contemplation for its end; and often, indeed, certain things, impossible to be executed, are called problems: but more properly that which is possible to be done, and neither exceeds, nor is deficient, is allotted an appellation of this kind; and the problem *exceeds*, which says, *to construct an equilateral triangle, having its vertical angle two thirds of one right*; for this is superfluous, and is added in vain: since it is a property inherent in every equilateral triangle. But of those which exceed, whatever are redundant with incongruous and non-existent symptoms, are called *impossibles*. But a *defective* problem (which is also called a *less problem*) is that which requires some addition, that it may be reduced from inordination into order and scientific bound, as if any one should say, *to constitute an isosceles triangle*: for this is mutilated and indeterminate, and requires some one who may subjoin, what kind of isosceles triangle, whether that which has its base greater than either of the equal sides; of that which has it less. Likewise, whether that which has the vertical angle double of each at the base, as a semiquadrangle; or that which has each of the angles at the base double of the vertical angle; or that which possesses these angles according to some other proportion, as triple or quadruple: for it is possible that it may be varied in infinite modes. From hence, therefore, it is manifest, that such things are properly denominated problems, ought to avoid indetermination, and not to be of the number of things capable of infinite variation; though such as these are also called problems, through an equivocation of the word problem. The first problem, therefore, of these elements, excels the rest in the manner we have explained; for it neither *exceeds*, nor is *deficient*; it is neither constructed in a variety, nor according to infinite modes; and such ought to be the conditions of that which is to be the element of the rest.