The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792)

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 6–12 (1792).]

PETITIONS or POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

That a terminated straight line may be produced to any length[129] in a straight line.

III.

And that a circle may be described from any centre, at any distance from that centre.

According to the opinion of Geminus, these three are necessarily placed among petitions, as well on account of their facility, as because they command us to do something. For this, *to draw a right line from every point, to every point*, follows the definition, which says, *that a line is the flux of a point*, and a right line *an indeclinable and inflexible flow.* If then we conceive a point to be moved with an uninclined, and the shortest motion, we shall fall upon another point, and the first petition will be produced, and we shall understand nothing various or difficult. But if when the right line itself is terminated by a point, we conceive its extremity moved with the shortest indeclinable motion, the second petition will arise from an easy and simple apprehension. But if we again imagine that the terminated right line abides according to its other extreme, but that it moves about that which abides according to the rest, the third petition will be produced; for the centre is the point which abides, but the interval the right line. Since the distance from the centre, from all points of the circumference, is always equal to the quantity of this interval. But if any one should doubt how we apply motion in geometrical concerns which have an immoveable existence; and how we

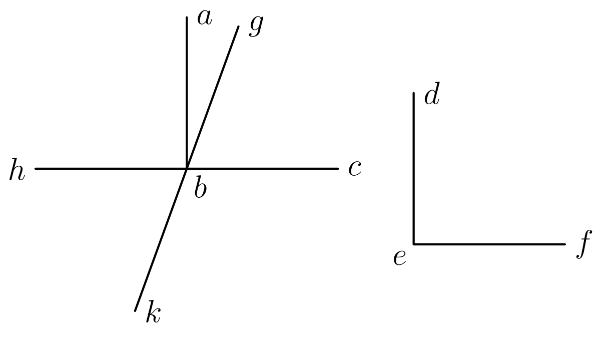
[129] [DRW—The clause "to any length" seems to be an addition of the translator, Thomas Taylor; The Greek text, in Friedlein's edition of Proclus's Commentaries (p. 185, 2–4) reads as follows: καὶ πεπερασμένην εὐθεῖαν κατὰ τὸ συνεχὲς ἐπ᾽ εὐθείας ἐκβαλεῖν, in accordance with Heiberg's edition of Euclid's *Elements*.]

can move impartibles, (since this is imposisble) we request him to call to mind what we have demonstrated in the beginning of these Commentaries. I mean that the reasons of things subsisting in the phantasy, describe there all the images of cogitation, of which cogitation itself possesses the reason: for an intellect of this kind is an unwritten, ultimate, and passive tablet. Hence it receives forms from another, accompanied with motion; but we must not understand a corporeal but imaginative motion, and must by no means admit that impartibles are moved with corporeal motions, but that they suffer imaginative progressions. For intellect, though impartible, is moved, yet not according to place, and the phantasy has a proper motion according to the impartible which it contains: but we only regarding corporeal motions, neglect those which are made in things destitute of interval. Impartibles, therefore, are pure from corporeal place, and external motions: but another species of motion, and another place congenial to such motions, is considered in their progressions. For, indeed, we should say, that a point also has position in the phantasy, and should not enquire how an impartible can abide, which is at the same time moved elsewhere, and comprehended by place. Since the place of things, with dimension, possesses itself dimension; but the place of impartibles is destitute of all dimension. The proper species therefore of geometrical concerns, are different from the things they produce; and the motion of bodies is different from that of the forms in the phantasy; and the place of partible is different from that of impartible natures; and it is requisite, by distinguishing these, neither to confound nor disturb the essences of things. But it appears that the first of these three petitions declares to us in images, how the *things which are*, are contained in their own impartible causes, and are terminated by their immaterial bound; and that previous to their constitution, they are on all sides comprehended in their indivisible embrace: for the points existing, a right line is drawn from the one to the other, is terminated by, and received between them. But the second indicates how *the things which are* by possessing proper causes proceed to all things, preserving in them a continuation not derived from the natures into which they proceed; but that through a cause of infinite power, they endeavour to permeate every where, with a never-failing progression. And the third petition shadows forth the manner in which these progressions return again to their proper principles: for the convolution of a point producing a circle, but moving about an abiding point, imitates a circular regression. But it is requisite to know, that every line cannot be infinitely produced, for the circle and cissoid, and all such as described figure, are incapable of this property; as likewise some which produce no figure. For the helix of one revolution cannot be infinitely produced, since it is constituted between two points; nor any other lines similarly formed. But neither is it possible to extend every line from every point, to every point; for every line cannot subsist between all points: and thus much for the three first petitions; let us now proceed to the rest.

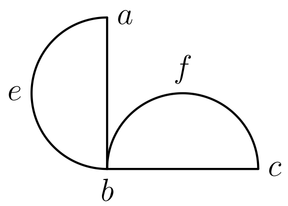
IV.

All right angles are equal to each other.

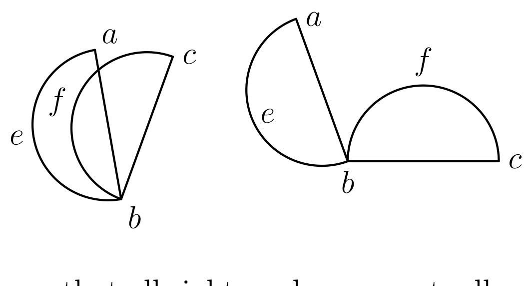
If the present petition is considered by us as manifest, and as requiring no demonstration, it is not a petition according to the opinion of Geminus, but an axiom; for it affirms a certain essential accident of right angles, not commanding us to perform any thing according to a simple conception. But neither is it a petition according to the division of Aristotle: for petition, according to his opinion, requires some demonstration. But if we should say it is demonstrable, and enquire after its demonstration, yet according to the opinion of Geminus, it ought not to be placed among petitions. The equality, therefore, of right angles, appears from our common conceptions; for since a right angle has the relation of unity or bound to the infinite increase and decrease of the angles on each side, it is equal with respect to every right angle, since we constitute the first right angle after this manner, by a right line making angles on each side of the right line on which it stands equal to each other; but if it be requisite to produce a linear demonstration of this, let there be two right angles, on $a\,b\,c$, the other $d\,e\,f$. I say that they are



equal; for if they are not equal, one of them must be greater, suppose the angle at b. If then the line $d\,e$ be adapted to the line $a\,b$, the line $e\,f$ shall fall within. Let it fall as $b\,g$, and let the line $b\,c$ be produced to h; because, then $a\,b\,c$ is a right angle, $a\,b\,h$ also shall be a right angle, and they shall be mutually equal to each other, from the tenth Definition: the angle $a\,b\,h$, therefore is greater than the angle $a\,b\,g$. Let again the line $g\,b$ be produced to k, because, therefore $a\,b\,g$ is a right angle; the successive angle $a\,b\,k$ shall be a right one, and consequently equal to $a\,b\,g$. Hence, the angle $a\,b\,h$, shall be less than the angle abg; but it was also greater, which is impossible; but this has been shewn by other expositors, and requires no great consideration. But Pappus very properly admonishes, the converse of this Petition is not true: I mean, that every things equal to a right angle, is a right angle; though if it be rectelinear [*sic.*], it is no doubt a right angle. But a curvilinear angle may also be exhibited equal to one that is right: for let there be conceived two equal right lines ab, and bc, making the angle at the point b, right; and



on them let the semicircles aeb, bfc, with a proper centre and interval be described; because, therefore, the semicircles are equal, they shall have a mutual congruence, and the angle eba, is equal to the angle fbc, and abf is common: the whole right angle, therefore, is equal to the lunular, i. e. to ebf, and yet the lunular is not a right angle. In the same manner, if the angle abc should be obtuse or acute, a lunular angle may be shewn equal to it (for this is that genus of curvilinear angles which agrees with such as are rectilinear), only this is to be observed, that in a right and obtuse angle, it is requisite to add the middle angle, which is contained by the line ab, and the circumference bf; but in an acute angle to take this away: for the right line cb, in these cases, cuts the circumference be. The truth of which, will be evident from the following figures:



And hence, it appears, that all right angles are mutually equal to each other, and that not every thing equal to a right angle, is consequently a right angle: for if it be not rectilinear, how can it be called right. But it is also manifest from this Petition, that angular rectitude is allied to equality, in the same manner as acuteness and obtuseness are related to inequality. For rectitude and equality, as also similitude, are of the same co-ordination, (for each exists under bound): but acuteness and obtuseness, as also dissimilitude, are of the same series with inequality. For they are all produced from *bound* and *infinite*. Hence some, regarding the quantity of angles, say, that a right angle is equal to a right: but others, considering their quality, affirm that one is similar to another. For similitude in qualities is the same as equality in quantities.

V.

> If a right line falling upon two right lines, makes the internal angles towards the same part less than two right, those right lines, if infinitely produced, shall coincide in that part, in which the angles less than two right, are placed.

This ought to be entirely blotted out from the number of Petitions, for it is a theorem including many doubts, which Ptolemy in one of his books proposes to solve; but it requires in its demonstration both many definitions and theorems; and Euclid also exhibits its converse as a theorem. But perhaps some, from an erroneous conception, may think that this should be placed among the petitions, as that which produces credibility of itself, respecting the inclination of right lines, on account of the diminution of two right angles. To such as these, Geminus rightly answers, that from the authors of this science, we learn not entirely to give credit to imaginative probabilities, for the purpose of accomplishing geometrical reasons: for it is similar (says Aristotle) to require demonstrations from a rhetorician, and patiently listen to a geometrician, disputing from probability. And Simmeas in the Phædo, says, "I know that those who demonstrate from appearances, are vain." Hence, in the present instance, it is true and necessary that right lines should incline, which right angles are diminished: but this, that the inclining lines, while they are more and more produced, should at length coincide, is probable, but not necessary, unless some reason demonstrates that this is true in right lines: for there are certain lines infinitely inclining, and never coinciding, and though this appears incredible and admirable, yet it is true, and has been observed in other forms of a line. Is it therefore possible that this can be accomplished in right lines which takes place in others? For before we procure conviction of this, from demonstration, the properties exhibited in other lines molest the phantasy by the contrary images they produce. But if the reasons doubting against the coincidence of lines are very strong, ought we not much more to expel this improbable and irrational supposition from our doctrine? And thus it appears that a demonstration is to be sought for of the present theorem, and that it is foreign from the property of Petitions: but how it is to be demonstrated, and by what reasons the objections urged against it are to be removed, we shall shew in our comment on the proposition, where it used by the institutor of the elements as manifest. For then it will be necessary to exhibit its evidence, since it does not present itself to our view with indemonstrable clearness, but becomes manifest through the medium of demonstration alone.