# The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792)

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[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 2, pp. 12–18 (1792).]

# AXIOMS.

I.

Things which are equal to the same, are equal to one another.

# II.

If equals be added to equals, the wholes are equal.

III.

If equals be taken from equals, the remainders are equal.

### $\mathrm{IV}.$

If equal are added to unequals, the wholes are unequal.

#### ν.

If equals be taken from unequals, the remainders are unequal.

#### VI.

Things which are double of the same, are equal to one another.

#### VII.

Things which are halves of the same, are equal to one another.

VIII.

Things which coincide with each other, are mutually equal.

#### IX.

The whole is greater than its part.

# Х.

Two right lines cannot comprehend space.

These are the things which, according to the opinion of all men, are called indemonstrable axioms, so far as their certainty is admitted by all, and no one disputes their evidence. For propositions also are often simply called axioms, of whatever kind they may be, whether they are immediately proper, or require some declaration; and the Stoics, indeed, are accustomed to call every simple enunciative speech an axiom: and when they write on dialectic arts, they say that they discourse on axioms. But some, distinguishing more accurately axioms from other propositions, give this appellation to a proposition immediate, and producing credibility of itself, on account of its evidence: as also Aristotle and geometricians themselves affirm. For, according to the opinion of these, an axiom is the same as a common conception. By no means, therefore, must we praise Apollonius the geometrician, who writ (as it appears) demonstrations of axioms, because he performs the very opposite to Euclid: for he, indeed, enumerates that which is demonstrable among Petitions; but Apollonius endeavours to find out demonstrations of indemonstrables. But these naturally differ from each other, and the genus of the sciences is different: I mean of the things which take place about immediate propositions, which are entirely subject to our knowle[d]ge, on account of their evidence; and of things which use demonstrations, which receive principles from them; and which, when received, they orderly employ in their proper conclusions. But that the demonstration of the first axiom, which Apollonius persuades himself he has invented, possesses a medium, not more known, but more dubious than the conclusion may be known by any one from a slight inspection. For let (says he) a be equal to b, and b to c, I say that a also is equal to c. For since a is equal to b, it occupies the same place as b. And because b is equal to c, it occupies the same place as c; and so a occupies the same place as c, they are therefore equal. Now in this demonstration it is requisite that two things must be previously assumed; one, that things occupying the same place, are mutually equal; but the other, that things occupying the same place, with the same thing, mutually occupy the



same place: but these are evidently more obscure than the present axiom. For it is proper to enquire how are thing[s], which fill the same place equal, according to the whole, or according to a part; or according to a figure of speech: hence we must by no means admit a transition to place,<sup>130</sup> which is

 $<sup>^{130}</sup>$ The nature of place has been a subject of much curious and deep speculation to the Peripatetic and Platonic philosophers, as may be seen in the very valuable Commentaries of Simplicius on Aristotle's Physics; so that Proclus does not affirm without reason, that place is more obscure than the natures it contains. But as the opinion of our philosopher, concerning place, is so admirably profound, subtle, and remarkable, I persuade myself the following translation from the fourth book of Simplicius on the Physics, containing his sentiments at large on this subject, will not be unacceptable to the liberal English reader. "Proclus (says Simplicius) having proved from the arguments of Aristotle, that place is neither matter nor form, concludes, that it is a certain interval:" after which, he reasons as follows. "This interval then, is either, nothing or something; and if nothing, local motion will consists in a transition from nothing to nothing; but all motion subsists according to something. But if it ought to be called something, it is either corporeal or incorporeal; and if incorporeal, and absurdity will ensue; for it is necessary that place should be equal to the thing placed. But how can body, and that which is incorporeal be equal? For equal is found in quantities, and especially in those of a similar kind, as lines are equal to lines, superficies to superficies, and bodies to bodies: place, therefore, is body, if it be a certain interval; but if body, it is either moveable or immeoveable; and if moveable, in whatever manner it may be moved, it must necessarily be moved according to place, so that place again will require another place, which is impossible, as was also evident to Aristotle and Theophrastus; for Aristotle says, that a vessel is a moveable place, but place an immoveable vessel, because place is naturally immoveable. But if it be immoveable, it is either indivisible, which cannot be divided by the bodies entering its receptacle, since one body cannot penetrate another; or it is divisible, as air and water are divided by the bodies entering into their yielding natures; but if it be divisible, the whole being dissected or divided, the divided parts will be moved on each side, and place will be the first mutable, since its parts are moved; but we have demonstrated that it is immoveable. Again, the parts being separated, we ask where that which is divided betakes itself; for there must be again given or investigated another interval, intervening between the divided parts, which may receive and be placed together with that which is divided; and this will be the case, in infinitum. Place, therefore, is an indivisible body; and if an indivisible body, either material, or destitute of matter: but if material it will not be indivisible, for it is requisite that all material bodies, when permeated by other material bodies, should be divided by them, as is the case with our bodies when they fall into water. But immaterials alone resist all division and this from a necessity of nature; for every body destitute of matter is void of passion; but every thing which is divided likewise suffers. Since division is a certain affectation of bodies, which extirpates and destroys their unity and connection; for that

more unknown than the natures it contains; for the invention of its essence is difficult and ambiguous. That we may avoid prolixity, therefore, all axioms are to be delivered as things immediate and self-manifest, since they are of themselves known and credible; for he who brings demonstration to things the most manifest, does not confirm their truth, but diminishes the evidence we possess in the untaught and innate conceptions of the soul: but this is to be received concerning axioms, as a jusgment of their peculiarity; and that all of them are of the common kind of the mathematical sciences; and that each of them is said to be verified, not only in magnitudes, but also in numbers, and motions, and times: and this indeed is necessary. For equal and unequal, the whole and part, and the more and the less, are common to discreet [sic.] and continued quantities. The contemplation, therefore, which is conversant with times and motions, numbers and magnitudes, requires all these, as things evident by their own intrinsic light; and in all of them both that is true, which says, things equal to the same, are equal to one another; as likewise each of the axioms we have assumed; but as they exist in common, each science uses them according to its proper matter, and one indeed, as in magnitudes; but another, as in numbers; and another, as in times; and after this manner in each science, the conclusions become peculiar and apposite, though the axioms are common. Besides, it is likewise requisite not to contract the number of these to the least, as is done by Heron, who only establishes three axioms; for this also is an axiom, the whole is greater than its *part*, and the geometrician every where assumes this in his demonstrations; as also, that things which mutually coincide, are equal; for this is employed

which is continuous, so far as continuous suffers no affection or molestation than section, which destroys and takes away its continuity. That we may therefore collect togehter what we have separately demonstrated, place is an immoveable indivisible body, destitute of matter. And if this be admitted, it is evident that it is a body by far less material than the rest, and indeed less than the matter contained in things which are moved. Hence, if light is the most simple of these (for fire is more incorporeal than the other elements, and fire is lucid) it will be manifest, that since light is the purest among the rest, light will be place. Conceive, therefore, two spheres, of which one is composed from many bodies, and the other of light alone, and let both be of equal builk; then, by establishing the sphere of light, together with the centre, and giving the composite sphere a revolution in the circumscribing sphere of light, you will perceive the world moved in immoveable light, and according to its whole extension, immoveable, similar to place, but moved according to its parts, because these are less than place." Now, from this demonstration of Proclus, it follows by a necessary consequence, since contraries are contained under the same genus that darkness, if it be any thing positive, is the most material of all bodies; and hence, the most material natures will participate the most of darkness, as indeed, is evident in the elements of earth and water. It likewise follows that whatever exists in perfect darkness, exists out of corporeal place, which, however paradoxical, is perhaps, no less true than wonderful to conceive.

with advantage in the solution of the fourth Proposition. Nor is it proper to join some with others, of which some are proper to the geometric matter, as that two right lines cannot comprehend space, (since axioms are, as we have said, of a common kind); but others are consequent to things established, as that which says, things double of the same, are equal. For this is consequent to the axiom, affirming, that if to equals you add equals, the wholes are equal, since things equal to the half, because they assume the half, become double to the same, and mutually equal, on account of an equal addition: and according to this reason, not only the doubles, but also the triples, and all multiples of the same quantity will appear equal. But with these axioms, Pappus says, that certain others are to be classed, as *if unequals are added* to equals, the excess of the whole, will be equal to the excess of the adjuncts. And on the contrary, if equals are added to unequals, the excess of the wholes is equal to the excess or difference of the unequals themselves. And these also are manifest from themselves, yet they may be made manifest as follows. Let a be equal to b, and add to each the unequals cd, but let c be greater than d by e, and the remainder be f; because, therefore, a is equal to b, and also f to d; a f will be equal to bd. For if equals are added to equals, the wholes



are equals: ac, therefore, exceeds bd, by e only, by which alone c exceeds d. Again, c and d are unequals, to which, let the equals a and b be added, and let e be the excess of c, above d, and the remainder be f; because, therefore a is equal to b, and f to d, af will be equal to bd; the whole, therefore, ac, will exceed bd, by e only, by which c also exceeds d. These, therefore, are consequent to the aforesaid axioms, and are, not undeservedly, in many copies, omitted. But whatever others he adds to these, have been previously assumed by definitions, to which they are consequent. As for example: that all the parts of a plane and a right line mutually agree; for things placed in their extremities, possess a nature of this kind; and that a point divides a line, but a line a superficies, and a superficies a solid. For all things are divided by the natures by which they are proximately bounded; and that infinite subsists in magnitudes, by addition and diminution, but accordingto capacity only, in both these respects: for every thing continuous may be infinitely divided and increased. But, as we have summarily spoken concerning these, it remains that we consider things consequent to principles; for thus far principles extend themselves. But of those who oppose geometry, some very much doubt concerning principles, endeavouring to shew that the terms have no subsistence, whose arguments, indeed, are known in common, who endeavour to take away all science, and, like hostile foes from a foreign region, demolish the fruits and fecundity of philosophy, as is the case with the Pyrrhonian philosophers; but others only propose to themselves the subversion of geometrical principles, as the Epicureans. Others, again, admitting the principles, affirm, that things consequent to the principles cannot be demonstrated, unless something else is granted, which was not previously assumed in the principles. Zeno exercised this mode of contradiction, who was a Sidonian by birth, but of the Epicurean sect, against whom Possidonius wrote an entire book, exhibiting the whole of his imbecile opinion; and thus much may suffice for the difference of opinions concerning principles. We shall shortly consider the troublesome objection of Zeno: but now, after we have briefly resumed the consideration of theorems and problems, their difference, and the divisions they receive, we shall proceed to an exposition of the things exhibited by the institutor of the elements, gathering the more beautiful observations upon the propositions found in the writings of the antients, and contracting the infinite prolixity of their discourses; but delivering such things as are more artificial, and full of methods producing science, dwelling more on an accurate treatise of things than on the variety of cases and assumptions, to which young men, for the most part, eagerly incline.