[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 338–339 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 41.]

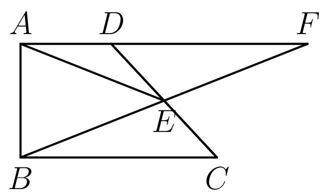
On this proposition Proclus (pp. 414, 15–415, 16), "by way of practice" (γυμνασίας ἕνεκα), considers the area of a *trapezium* (a quadrilateral with one one pair of opposite sides parallel) in comparison with that of the triangles in the same parallels and having the greater and less of the parallel sides of the trapezium for bases respectively, and proves that the trapezium is less than double of the former triangle and more than double of the latter.

He next (pp. 415, 22–416, 14) proves the proposition that,

If a triangle be formed by joining the middle point of either of the non-parallel sides to the extremities of the opposite side, the area of the trapezium is always double of that of the triangle.

Let $ABCD$ be a trapezium in which AD, BC are the parallel sides, and E the middle point of one of the non-parallel sides, say DC.

Join EA, EB and produce BE to meet AD produced in F.



Then the triangles BEC, FED have two angles equal respectively, and one side CE equal to one side DE;
therefore the triangles are equal in all respects. [I. 26]

Add to each the quadrilateral $ABED$;
therefore the trapezium $ABCD$ is equal to the triangle ABF,
that is, to twice the triangle AEB, since BE is equal to EF. [I. 38]

The three properties proved by Proclus may be combined in one enunciation thus:

If a triangle be formed by joining the middle point on one side of a trapezium to the extremities of the opposite side, the area of the trapezium is (1) greater than, (2) equal to, or (3) less than, double the area of the triangle according as the side the middle point of which is taken is (1) the greater of the parallel sides, (2) either of the non-parallel sides, or (3) the lesser of the parallel sides.