[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 325–326 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 34.]

1. It is to be observed that, when parallelograms have to be mentioned for the first time, Euclid calls them "**parallelogrammic areas**" or, more exactly, "parallelogram" areas (παραλληλόγραμμα χωρία). The meaning is simply areas bounded by parallel straight lines with the further limitation placed on the term by Euclid that only *four-sided* figures are so called, although of course there are certain regular polygons which have opposite sides parallel, and which therefore might be said to be areas bounded by parallel straight lines. We gather from Proclus (p. 393) that the word "parallelogram" was first introduced by Euclid, that its use was suggested by I. 33, and that the formation of the word παραλληλόγραμμος (parallel-lined) was analogous to that of εὐθύγραμμος (straight-lined or rectilineal).

17, 18, 40. **DCB** and 36. **DC**, **CB**. The Greek has in these places "*BCD*" and "*CD*, *BC*" respectively. Cf. note on I. 33, lines 15, 18.

After specifying the particular kinds of parallelograms (squares and rhombi) in which the diagonals bisect the angles which they join, as well as the areas, and those (rectangles and rhomboids) in which the diagonals do not bisect the angles, Proclus proceeds (pp. 390 sqq.) to analyse this proposition with reference to the distinction in Aristotle's *Anal. Post.* (I. 4, 5, 73 a 21–74 b 4) between attributes which are only predicable of every individual thing (κατὰ παντός) in a class and those which are true of it *primarily* (τούτου πρώτου) and *generally* (καθόλου). We are apt, says Aristotle, to mistake a proof κατὰ παντός for a proof τούτου πρώτου καθόλου because it is either impossible to find a higher generality to comprehend all the particulars of which the predicate is true, or to find a name for it. (Part of this passage of Aristotle has been quoted above in the note on I. 32, pp. 319–320.)

Now, says Proclus, adapting Aristotle's distinction to *theorems*, the present proposition exhibits the distinction between theorems which are *general* and theorems which are *not general.* According to Proclus, the first part of the propostion stating that the opposite sides and angles of a parallelogram are equal is *general* because the property is only true of parallelograms; but the second part which asserts that the diameter bisects the area is *not general* because it does not include all the figures of which this property is true, e.g. circles and ellipses. Indeed, says Proclus, the first attempts upon problems seem usually to have been of this partial character (μερικώτεραι), and generality was only attained by degrees. Thus "the ancients, after investigating the fact that the diameter bisects an ellipse, a circle, and a parallelogram respectively, proceeded to investigate what was common to these cases," though “it is difficult to show what is common to an ellipse, a circle and a parallelogram.”

I doubt whether the supposed distinction between the two parts of the proposition, in point of “generality,” can be sustained. Proclus himself admits that it is presupposed that the subject of the proposition is a *quadrilateral*, because there are other figures (e.g., regular polygons of an even number of sides) besides parallelograms which have their opposite sides and angles equal; therefore the second part of the theorem is, in this respect, no more *general* than the other, and, if we are entitled to the tacit limitation of the theorem to quadrilaterals in one part, we are equally entitled to it in the other.

It would almost appear as though Proclus had drawn the distinction for mere purpose of alluding to investigations by Greek geometers on the general subject of *diameters* of all sorts of figures; and it may have been these which brought the subject to the point at which Apollonius could say in the first definitions at the beginning of his *Conics* that “In *any bent line*, such as is in one plane, I give the name *diameter* to any straight line which, being drawn from the bent line, bisects all the straight lines (chords) drawn in the line parallel to any straight line.” The term *bent line* (καμπύλη γραμμή) includes, e.g., in Archimedes, not only curves, but any composite line made up of straight lines and curves joined together in any manner. It is of course clear that either diagonal of a parallelogram bisects all lines drawn within the parallelogram parallel to the other diagonal.

An-Nairīzī gives after I. 31 a neat construction for dividing a straight line into any number of equal parts (ed. Curtze, p.74, ed. Besthorn-Heiberg, pp.141–3) which requires only one measurement repeated, together with the properties of parallel lines including I. 33, 34. As I. 33, 34 are assumed, I place the problem here. The particular case taken is the problem of dividing a straight line into *three* equal parts.

Let AB be the given straight line. Draw AC, BD at right angles to it on opposite sides.

An-Nairīzī takes AC, BD of the same length and then bisects AC at E and BD at F. But of course it is even simpler to measure AE, EC along one perpendicular equal and of any length, and BF, FD along the other also equal and of the same length.

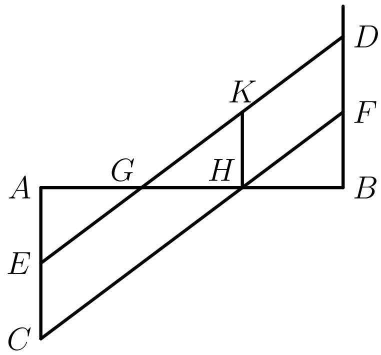
Join ED, CF meeting AB in G, H respectively.

Then shall AG, GH, HB all be equal.

Draw HK parallel to AC, or at right angles to AB.

Since now EC, FD are equal and parallel, ED, CF are equal and parallel. [I. 33]

And HK was drawn parallel to AC.



Therefore $ECHK$ is a parallelogram; whence KH is equal as well as parallel to EC, and therefore to EA.

The triangles EAG, KHG have now two angles respectively equal and the sides AE, HK equal.

Thus the triangles are equal in all respects, and

AG is equal to GH.

Similarly the triangles KHG, FBH are equal in all respects, and GH is equal to HB.

If now we wish to extend the problem to the case where AB is to be divided into n parts, we have only to measure $(n-1)$ successive equal lengths along AC and $(n-1)$ successive lengths, each equal to the others, along BD. Then join the first point arrived at on AC to the last point on BD, the second on AC to the last but one on BD, and so on; and the joining lines cut AB in points dividing it into n equal parts.