[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 314–315 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 30.]

20. The usual *conclusion* in general terms ("Therefore etc.") repeating the enunciation is, curiously enough, wanting at the end of this proposition.

The proposition is, as De Morgan points out, the *logical* equivalent of "Playfair's" axiom. Thus, if X denote "pairs of straight lines intersecting one another," Y "pairs of straight lines parallel to one and the same straight line," we have

$$\text{No } X \text{ is } Y,$$

and it follows logically that

$$\text{No } Y \text{ is } X.$$

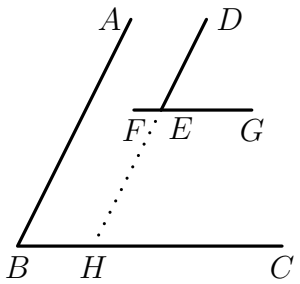
De Morgan adds that a proposition is much wanted about parallels (or perpendiculars) to two straight lines respectively making the same angles with one another as the latter do. The proposition may be enunciated thus:

If the sides of one angle be respectively (1) parallel or (2) perpendicular to the sides of another angle, the two angles are either equal or supplementary.

(1) Let DE be parallel to AB and GEF parallel to BC.

To prove that the angles ABC, DEG are equal and the angles ABC, DEF supplementary.

Produce DE to meet BC in H.



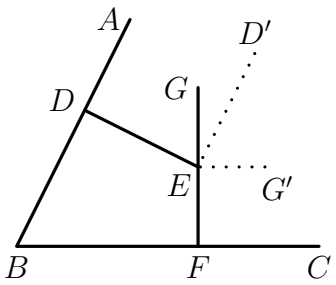
Then [I. 29] the angle DEG is equal to the angle DHC,

and the angle ABC is equal to th angle DHC.

Therefore the angle DEF is equal to the angle ABC; whence also the angle DEF is supplementary to the angle ABC.

(2) Let ED be perpendicular to AB, and GEF perpendicular to BC.

To prove that the angles ABC, DEG are equal, and the angles ABC, DEF supplementary.



Draw ED' at right angles to ED on the side of it opposite to B, and draw EG' at right angles to EF on the side of it opposite to B.

Then, since the angles BDE, DED', being right angles, are equal,

ED' is parallel to BA. [I. 27]

Similarly EG' is parallel to BC.

Therefore [Part (1)] the angle $D'EG'$ is equal to the angle ABC.

But, the right angle DED' being equal to the right angle GEG', if the common angle GED' be subtracted,

the angle DEG is equal to the angle $D'EG'$.

Therefore the angle DEG is equal to the angle ABC; and hence the angle DEF is supplementary to the angle ABC.