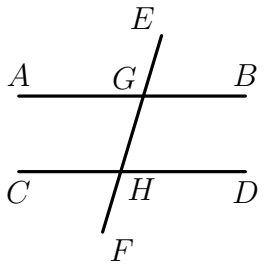
[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 310–311 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 28.]

One criterion of parallelism, the equality of alternate angles, is given in I. 27; here we have two more, each of which is easily reducible, and is actually reduced, to the other.

Proclus observes (pp. 258–9) that Euclid could have stated six criteria as well as three, by using, in addition, other pairs of angles in the figure (not adjacent) of which it could be predicated that the two angles are equal or that their sum is equal to two right angles. A natural division is to consider, first the pairs which are on the same side of the transversal, and secondly the pairs which are on different sides of it.



Taking (1) the possible pairs on the *same* side, we may have a pair consisting of

(a) two internal angles, viz. the pairs (BGH, GHD) and (AGH, GHC);

(b) two external angles, viz. the pairs (EGB, DHF) and (EGA, CHF);

(c) one external and one internal angle, viz. the pairs (EGB, GHD), (FHD, HGB), (EGA, GHC) and (FHC, HGA).

And (2) the possible pairs on *different* sides of the transversal may consist respectively of

(a) two internal angles, viz. the pairs (AGH, GHD) and (CHG, HGB);

(b) two external angles, viz. the pairs (AGE, DHF) and (EGB, CHF);

(c) one external and one internal, viz. the pairs (AGE, GHD), (EGB, GHC), (FHC, HGB) and (FHD, HGA).

The angles are equal in the pairs (1) (c), (2) (a) and (2) (b), and their sum is equal to two right angles in the case of the pairs (1) (a), (1) (b) and (2) (c). For his criteria Euclid selets the cases (2) (a) [I. 27] and (1) (c), (1) (a) [I. 28], leaving out the other three, which are of course equivalent but are not quite so easily expressed.

From Proclus' note on I. 28 (p. 361) we learn that one Aigeias (? Aineias) of Hierapolis wrote and epitome or abridgment of the *Elements*. This seems to be the only mention of this editor and his work; and they are only mentioned as having combined Eucl. I. 27, 28 into one proposition. To do this, or to make the three hypotheses the subject of *three* separate theorems, would, Proclus thinks, have been more natural than to deal with them, as Euclid does, in two propositions. Proclus has no suggestion for explaining Euclid's arrangement unless the ground were that I. 27 deals with angles on different sides, I. 28 with angles on one and the same side, of the transversal. But may not the reason have been one of convenience, namely that the criterion of I. 27 is that actually used to prove parallelism, and is moreover the basis of the construction of parallels in I. 31, while I. 28 only reduces the other two hypotheses to that of I. 27, so that precision of reference, as well as clearness of exposition, is better secured by the arrangement adopted?