[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 300–301 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 25.]

De Morgan points out that this proposition (as also I. 8) is a purely *logical* consequence of I. 4 and I. 24 in the same way as I. 19 and I. 6 are purely *logical* consequences of I. 18 and I. 5. If a, b, c denote the sides, A, B, C the angles opposite to them in a triangle ABC, and a', b', c', A', B', C' the sides and opposite angles respectively in a triangle $A'B'C'$, I. 4 and I. 24 tell us that, b, c, being respectively equal to b', c',

(1) if A is equal to A', then a is equal to a',
(2) if A is less than A', then a is less than a',
(3) if A is greater than A', then a is greater than a';

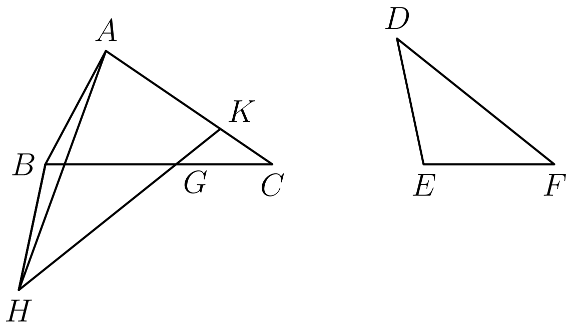
and it follows *logically* that,

(1) if a is equal to a', the angle A is equal to the angle A', [I. 8]
(2) if a is equal to a', A is less than A',
(3) if a is greater than a', A is greater than A'. } [I. 25]

Two alternative proofs of this theorem are given by Proclus (pp. 345–7), and they are both interesting. Moreover both are *direct.*

I. Proof by Menelaus of Alexandria.

Let ABC, DEF be two triangles having the two sides BA, AC equal to the two sides ED, DF, but the base BC greater than the base EF.



Then shall the angle at A be greater than the angle at D.

From BC cut off BG equal to EF. At B, on the straight line BC, make the angle GBH (on the side of BG remote from A) equal to the angle FED.

Make BH equal to DE; join HG, and produce it to meet AC in K. Join AH.

Then, since the two sides GB, BH are equal to the two sides FE, ED respectively,

and the angles contained by them are equal,

HG is equal to DF or AC,

and the angle BHG is equal to the angle EDF.

Now HK is greater than HG or AC,

and *a fortiori* greater than AK;

therefore the angle KAH is greater than the angle KHA.

And, since AB is equal to BH,

the angle BAH is equal to the angle BHA.

Therefore, by addition,

the whole angle BAC is greater than the whole angle BHG,

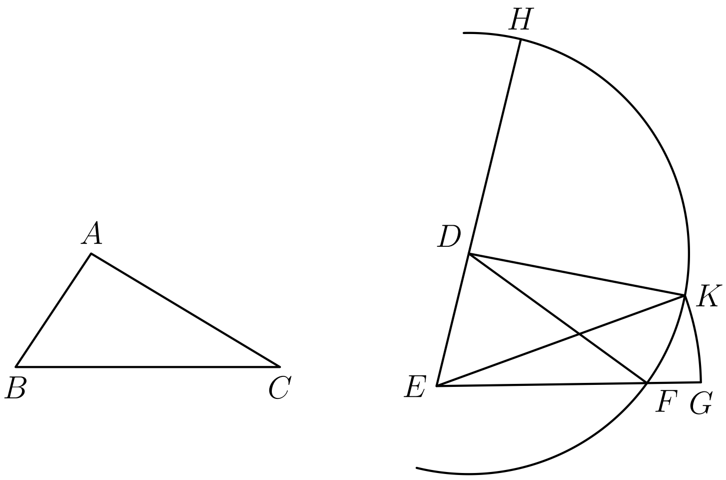
that is, greater than the angle EDF.

II. Heron's Proof

Let the triangles be given as before.

Since BC is greater than EF, produced EF to G so that EG is equal to BC.

Produce ED to H so that DH is equal to DF. The circle with centre D and radius DF will then pass through H. Let it be described, as FKH.



Now, since BA, AC are together greater than BC,

and BA, AC are equal to ED, DH respectively,

while BC is equal to EG,

EH is greater than EG.

Therefore the circle with centre E and radius EG will cut EH, and therefore will cut the circle already drawn. Let it cut that circle in K, and join DK, KE.

Then, since D is the centre of the circle FKH,

DK is equal to DF or AC.

Similarly, since E is the centre of the circle KG,

EK is equal to EG or BC.

And DE is equal to AB.

Therefore the two sides BA, AC are equal to the two sides ED, DK respectively;

and the base BC is equal to the base EK;

therefore the angle BAC is equal to the angle EDK.

Therefore the angle BAC is greater than the angle EDF.