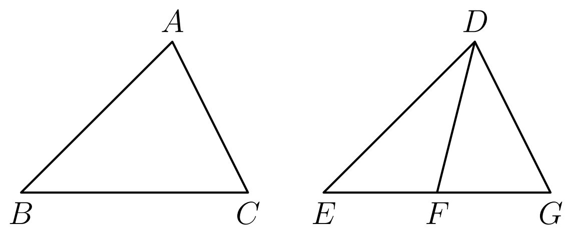
[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 297–299 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 24.]

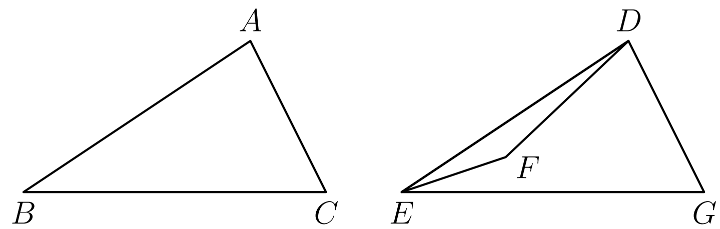
10. I have naturally left out the well-known words added by Simson in order to avoid the necessity of considering three cases: "Of the two sides DE, DF let DE be the side which is not greater than the other." I doubt whether Euclid could have been induced to insert the words himself, even if it had been represented to him that their omission meant leaving two possible cases out of consideration. His habit and that of the great Greek geometers was, not to set out all possible cases, but to give as a rule one case, generally the most difficult, as here, and to leave the others to the reader to work out for himself. We have already seen one instance in I. 7.

Proclus of course gives the other two cases which arise if we do not first provide that DE is not greater than DF.

(1) In the first case G may fall on EF produced, and it is then obvious that EG is greater than EF.



(2) In the second case EG may fall below EF.



If so, by I. 21, DF, FE are together less than DG, GE.

But DF is equal to DG; therefore EF is less than EG, i.e. than BC.

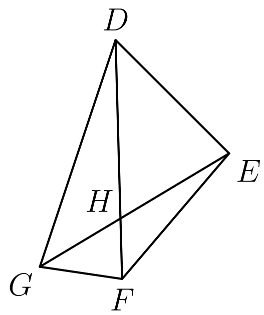
These two cases are therefore decidedly simpler than the case taken by Euclid as typical, and could well be left to the ingenuity of the learner.

If however after all we prefer to insert Simson's words and avoid the latter two cases, the proof is not complete unless we show that, with his assumption, F must, in the figure of the proposition, fall *below* EG.

De Morgan would make the following proposition precede: *Every straight line drawn from the vertex of a triangle to the base is less than the greater of the two sides, or than either if they are equal*, and he would prove it by means of the proposition relating to perpendicular and obliques given above, p. 291.

But it is easy to prove directly that F falls below EG, if DE is not greater than DG, by the method employed by Pfleiderer, Lardner and Todhunter.

Let DF, produced if necessary, meet EG in H.

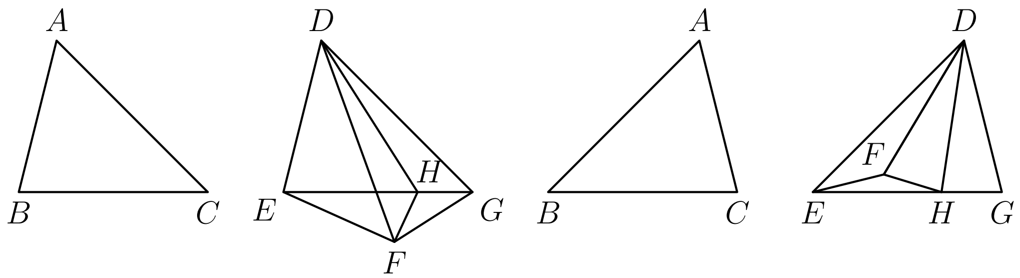


Then the angle DHG is greater than the angle DEG; [I. 16]
and the angle DEG is not less than the angle DGE; [I. 18]
therefore the angle DHG is greater than the angle DGH.
Hence DH is less than DG, [I. 19]
and therefore DH is less than DF.

Alternative proof.

Lastly, the modern alternative proof is worth giving.

Let DH bisect the angle FDG (after the triangle DEG has been made equal in all respects to the triangle ABC, as in the proposition), and let DH meet EG in H. Join HF.



Then, in the triangles FDH, GDH,

the two sides FD, DH are equal to the two sides GD, DH,

and the included angles FDH, GDH are equal;

therefore the base HF is equal to the base HG.

Accordingly EG is equal to the sum EH, HF;

and EH, HF are together greater than EF; [I. 20]

therefore EG, or BC, is greater than EF.

Proclus (p. 339, 11 sqq.) answers by anticipation the possible question that might occur to any one on this proposition, viz. why does Euclid not compare the areas of the triangles as he does in I. 4? He observes that inequality of the areas does not follow from the inequality of the angles contained by the equal sides, and that Euclid leaves out all reference to the question both for this reason and because the areas cannot be compared without the help of the theory of parallels. "But if," says Proclus, "we must anticipate what is to come and make our comparison of the areas at once, we assert that (1) *if the angles* A, D—supposing that our argument proceeds with reference to the figure in the proposition—*are (together) equal to two right angles, the triangles are proved equal*, (2) *if greater than two right angles, that triangle which has the greater angle is less, and* (3) *if they are less, greater*." Proclus then gives the proof, but without any reference to the source from which he quoted the proposition. Now an-Nairīzī adds a similar proposition to I. 38, but definitely attributes it to Heron. I shall accordingly give it in the place where Heron put it.