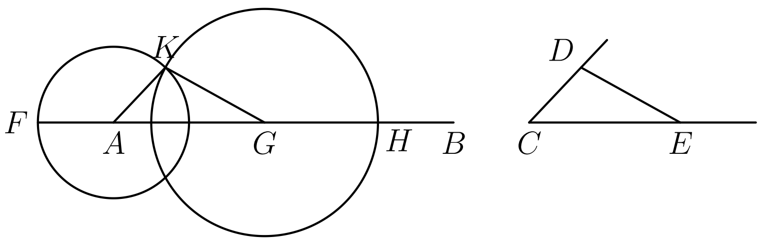
[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 295–296 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 23.]

This problem was, according to Eudemus (see Proclus, p. 333, 5), "rather the discovery of Oenopides," from which we must apparently infer, not that Oenopides was the first to find any solution of it, but that it was he who discovered the particular solution given by Euclid. (Cf. Bretschneider, p. 65.)

The editors do not seem to have noticed the fact that the construction of the triangle assumed in this proposition is not exactly the construction given in I. 22. We have here to construct a triangle on a certain finite straight line AG as base; in I. 22 we have only to construct a triangle with sides of a given length without any restriction as to how it is to be placed. Thus in I. 22 we set out any line whatever and measure successively three lengths along it beginning from the given extremity, and what we must regard as the base is the intermediate length, not the length beginning at the given extremity of the straight line arbitrarily set out. Here the base is a given straight line abutting at a given point. Thus the construction has to be modified somewhat from that of the preceding proposition. We must measure AG along AB so that AG is equal to CE (or CD), and GH along GB equal to DE; and then we must produce BA, in the opposite direction, to F, so that AF is equal to CD (or CE, if AG has been made equal to CD).

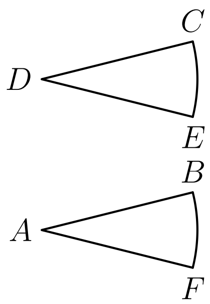


Then, by drawing circles (1) with centre A and radius AF, (2) with centre G and radius GH, we determine K, one of their points of intersection, and we prove that the triangle KAG is equal in all respects to the triangle DCE, and then that the angle at A is equal to the angle DCE.

I think that Proclus must (though he does not say so) have felt the same difficulty with regard to the use in I. 23 of the result of I. 22, and that this is probably the reason why he gives over again the construction which I have given above, with the remark (p. 334, 6) that "you may obtain the construction of the triangle in a more instructive manner (διδασκαλικώτερον) as follows."

Proclus objects to the procedure of Apollonius in constructing an angle under the same conditions, and certainly, if he quotes Apollonius correctly, the latter's exposition must have been somewhat slipshod.

"He takes an angle CDE at random," says Proclus (p. 335, 19 sqq.), "and a straight line AB, and with centre D and distance CD describes the circumference CE, and in the same way with centre A and distance AB the circumference FB. Then, cutting off FB equal to CE, he joins AF. And he declares that the angles A, D standing on equal circumferences are equal."



In the first place, as Proclus remarks, it should be premised that AB is equal to CD in order that the circles may be equal; and the use of Book III. for such an elementary construction is objectionable. The omission to state that AB must be taken equal to CD was no doubt a slip, if it occurred. And, as regards the equal angles "standing on equal circumferences," it would seem possible that Apollonius said this in *explanation*, for the sake of brevity, rather than by way of proof. It seems to me probable that his construction was only given from the point of view of *practical*, not theoretical, geometry. It really comes to the same thing as Euclid's except that DC is taken equal to DE. For cutting off the arc BF equal to the arc CE can only be meant in the sense of measuring the *chord* CE, say, with a pair of compasses, and then drawing a circle with centre B and radius equal to the chord CE. Apollonius' direction was therefore probably intended as a practical short cut, avoiding the actual drawing of the chords CE, BF, which, as well as a proof of the equality in all respects of the triangles CDE, BAF, would be required to establish *theoretically* the correctness of the construction.