[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 287–288 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 20.]

It was the habit of the Epicureans, says Proclus (p. 322), to ridicule this theorem as being evident even to an ass and requiring no proof, and their allegation that the theorem was "known" (γνώριμον) even to an ass was based on the fact that, if fodder is placed at one angular point and the ass at another, he does not, in order to get to his food, traverse the two sides of the triangle but only the one side separating them (an argument which makes Savile exclaim that its authors were "digni ipsi, qui cum Asino foenum essent," p. 78). Proclus replies truly that a mere perception of the truth of the theorem is a different thing from a scientific proof of it and a knowledge of the reason *why* it is true. Moreover, as Simson says, the number of axioms should not be increased without necessity.

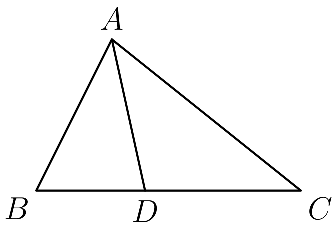
Alternative Proofs.

Heron and Porphyry, we are told (Proclus, pp. 323–6), proved this theorem in different ways as follows, without producing one of the sides.

First proof.

Let ABC be the triangle, and let it be required to prove that the sides BA, AC are greater than BC.

Bisect the angle BAC by AD meeting BC in D.



Then, in the triangle ABD,

the exterior angle ADC is greater than the interior and opposite angle BAD, [I. 16]

that is, greater than the angle DAC.

Therefore the side AC is greater than the side CD. [I. 19]

Similarly we can prove that AB is greater than BD.

Hence, by addition, BA, AC are greater than BC.

Second proof.

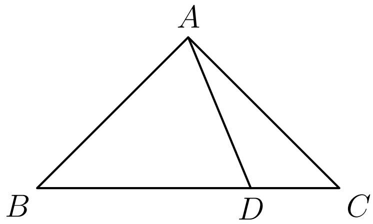
This, like the first proof, is direct. There are several cases to be considered.

(1) If the triangle is *equilateral*, the truth of the proposition is obvious.

(2) If the triangle is *isosceles*, the proposition needs no proof in the case (a) where each of the equal sides is greater than the base.

(b) If the base is greater than either of the other sides, we have to prove that the sum of the two equal sides is greater than the base. Let BC be the base in such a triangle.

Cut off from BC a length BD equal to AB, and join AD.



Then, in the triangle ADB, the exterior angle ADC is greater than the interior and opposite angle BAD. [I. 16]

Similarly, in the triangle ADC, the exterior angle ADB is greater than the interior and opposite angle CAD.

By addition, the two angles BDA, ADC are together greater than the two angles BAD, DAC (or the whole angle BAC).

Subtracting the equal angles BDA, BAD, we have the angle ADC is greater than the angle CAD.

It follows that AC is greater than CD; [I. 19]
and, adding the equals AB, BD respectively, we have BA, AC together greater than BC.

(3) If the tringle be *scalene*, we can arrange the sides in order of length. Suppose BC the greatest, AB the intermediate and AC the least side. Then it is obvious that AB, BC are together greater than AC, and BC, CA together greater than AB.

It only remains therefore to prove that CA, AB are together greater than BC.

We cut off from BC a length BD equal to the adjacent side, join AD, and proceed exactly as in the above case of the isosceles triangle.

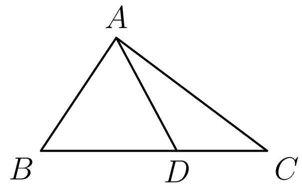
Third proof.

This proof is by *reductio ad absurdum*.

Suppose that BC is the greatest side and, as before, we have to prove that BA, AC are greater than BC.

If they are not, they must be either equal to or less than BC.

(1) Suppose BA, AC are together equal to BC.



From BC cut off BD equal to BA, and join AD.

It follows from the hypothesis that DC is equal to AC.

Then since BA is equal to BD, the angle BDA is equal to the angle BAD.

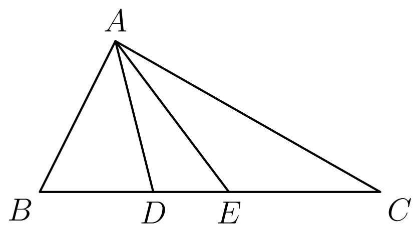
Similarly, since AC is equal to CD, the angle CDA is equal to the angle CAD.

By addition, the angles BDA, ADC are together equal to the whole angle BAC.

That is, the angle BAC is equal to two right angles: which is impossible.

(2) Suppose BA, AC are together less than BC.

From BC cut off BD equal to BA, and from CB cut off CE equal to CA. Join AD, AE.



In this case, we prove in the same way that the angle BDA is equal to the angle BAD, and the angle CEA to the angle CAE.

But addition, the sum of the angles BDA, AEC is equal to the sum of the angles BAD, CAE.

Now, by I. 16, the angle BDA is greater than the angle DAC, and therefore, *a fortiori*, greater than the angle EAC.

Similarly the angle AEC is greater than the angle BAD.

Hence the sum of the angles BDA, AEC is greater than the sum of the angles BAD, EAC.

But the former sum was also equal to the latter: which is impossible.