

[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 283–284 (1925).]

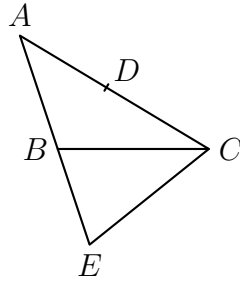
[Heath's commentary on Euclid, *Elements*, Book I, Proposition 18.]

In the enunciation of this proposition we have ὑποτείνειν (“subtend”) used with the simple accusative instead of the more usual ὑπό with accusative. The latter construction is used in the enunciation of I. 19, which otherwise only differs from that of I. 18 in the order of the words. The point to remember in order to distinguish the two is that the *datum* comes first and the *quaesitum* second, the *datum* being in this proposition the greater *side* and in the next the greater *angle*. Thus the enunciations are (I. 18) παντὸς τριγώνου ἡ μείζων πλευρὰ τὴν μείζονα γωνίαν ὑποτείνει and (I. 19) παντὸς τριγώνου ὑπὸ τὴν μείζονα γωνίαν ἡ μείζων πλευρὰ ὑποτείνει. In order to keep the proper order in English we must use the passive of the verb in I. 19. Aristotle quotes the result of I. 19, using the exact wording, ὑπὸ γὰρ τὴν μείζω γωνίαν ὑποτείνει (*Meteorologica* III. 5, 376 a 12).

“In order to assist the student in remembering which of these two propositions [I. 18, 19] is demonstrated directly and which indirectly, it may be observed that the order is similar to that in I. 5 and I. 6” (Todhunter).

An alternative proof of I. 18 given by Porphyry (see Proclus, pp. 315, 11–316, 13) is interesting. It starts by supposing a length equal to AB cut off from the other end of AC ; that is CD and not AD is made equal to AB .

Produce AB to E so that BE is equal to AD , and join EC .



Then, since AB is equal to CD , and BE to AD ,

AE is equal to AC .

Therefore the angle AEC is equal to the angle ACE .

Now the angle ABC is greater than the angle AEC ,

and therefore greater than the angle ACE .

Hence, *a fortiori*, the angle ABC is greater than the angle ACB .