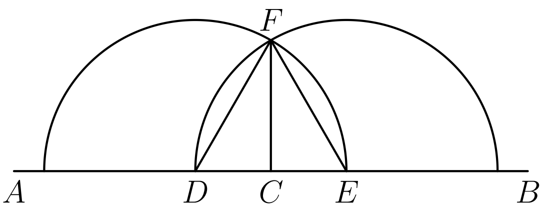
[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 269–270 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 11.]

10. **let CD be made equal to CD**. The verb is κείσθω which, as well as the other parts of κεῖμαι, is constantly used for the passive of τίθημι "to *place*"; and the latter word is constantly used in the sense of *making*, e.g., one straight line equal to another straight line.

De Morgan remarks that this proposition, which is "to bisect the angle made by a straight line and its continuation" [i.e., a *flat* angle], should be a particular case of I. 9, the constructions being the same. This is certainly worth noting though I doubt the advantage of rearranging the propositions in consequence.

Apollonius gave a construction for this proposition (see Proclus, p. 282, 8) differing from Euclid's in much the same way as his construction for bisecting a straight line differed from that of I. 10. Instead of assuming an equilateral triangle drawn without repeating the process of I. 1, Apollonius takes D and E equidistant from C as in Euclid, and then draws circles in the manner of I. 1 meeting at F. This necessitates proving again that DF is equal to FE;



whereas Euclid's asumption of the construction of I. 1 in the words "let the equilateral triangle FDE be constructed" enables him to dispense with the drawing of circles and with the proof that DF is equal to FE at the same time. While however the substitution of Apollonius' constructions for I. 10 and 11 would show faulty arrangement in a theoretical treatise like Euclid's, they are entirely suitable for what we call *practical* geometry, and such may have been Apollonius' object in these constructions and in his alternative for I. 23.

Proclus gives a construction for drawing a straight line at right angles to another straight line but from *one end* of it, instead of from an intermediate point on it, being supposed (for the sake of argument) that we are not permitted to *produce* the straight line. In the commentary of an-Nairīzī (ed. Besthorn-Heiberg, pp. 73–4; ed. Curtze pp. 54-5) this construction is attributed to Heron.

Let it be required to draw from A a straight line at right angles to AB.

On AB take any point C, and in the manner of the proposition draw CE at right angles to AB.

From CE cut off CD equal to AC, bisect the angle ACE by the straight line CF, [I. 9]
and draw DF at right angles to CE meeting CF in F. Join FA.

Then the angle FAC will be a right angle.

For, since, in the triangles ACF, DCF, the two sides DC, CF respectively, and the included angles ACF, DCF are equal,

the triangles are equal in all respects. [I. 4]

Therefore the angle at A is equal to the angle at D, and is accordingly a right angle.