[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 262–264 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 8.]

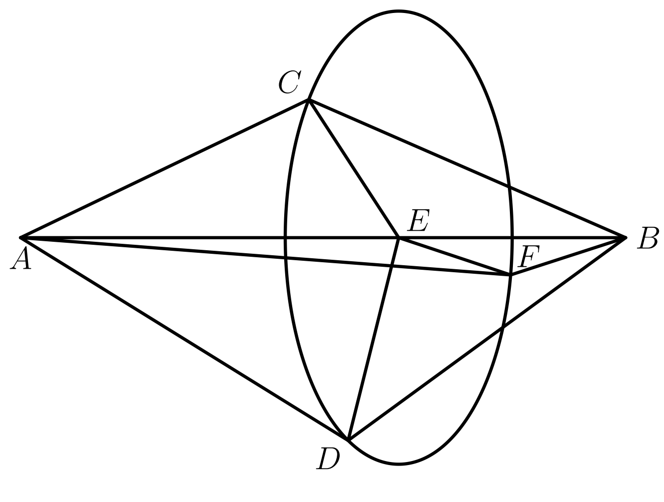
19. **BA, AC.** The text has here "BA, CA."

21. **fall beside them.** The Greek has the future, παραλλάξουσι. παραλλάττω means "to pass by without touching," "to miss" or "to deviate."

As pointed out above (p. 257) I. 8 is a *partial* converse of I. 4.

It is to be observed that in I. 8 Euclid is satisfied with proving the equality of the vertical angles and does not, as in I. 4, add that the triangles are equal, and the remaining angles are equal respectively. The reason is no doubt (as pointed out by Proclus and by Savile after him) that, when once the vertical angles are proved equal, the rest follows from I. 4, and there is no object in proving again what has been proved already.

Aristotle has an allusion to the theorem of this proposition in *Meteorologica* III. 3, 373 a 5–16. He is speaking of the rainbow and observes that, if equal rays be reflected from one and the same point to one and the same point, the points at which reflection takes place are on the circumference of a circle. "For let the broken lines ACB, AFB, ADB be all reflected from the point A to the point B (in such a way that) AC, AF, AD are all equal to one another, and the lines (terminating) at B i.e. CB, FB, DB, are likewise all equal; and let AEB be joined. It follows that *the triangles are equal*; for they are upon the equal (base) AEB."



Heiberg (*Mathematisches zu Aristoteles*, p. 18) thinks that the form of the conclusion quoted is an indication that in the corresponding proposition to Eucl. I. 8, as it lay before Aristotle, it was maintained that the *triangles* were equal, and not only the angles, and "we see here therefore, in a clear example, how the stones of the ancient fabric were recut for the rigid structure of his *Elements*." I do not, however, think that this inference from Aristotle's language as to the form of the pre-Euclidean proposition is safe. Thus if we, nowadays, were arguing from the data in the passage of Aristotle, we should doubtless infer directly that the triangles are equal in all respects, quoting I. 8 alone. Besides, Aristotle's language is rather careless, as the next sentences of the same passage show. "Let perpendiculars," he says, "be drawn to AEB from the angles CE from C, FE from F and DE from D. These, then, are equal; for they are all in equal triangles, and in one plane; for all of them are perpendicular to AEB, and they meet at one point E. Therefore the (line) drawn (through C, F, D) will be a circle, and its centre (will be) E." Aristotle should obviously have proved that the three perpendiculars *will* meet at one point E on AEB before he spoke of drawing the perpendiculars CE, FE, DE. This of course follows from their being "in equal triangles" (by means of Eucl. I. 26); and then, from the fact that the perpendiculars meet at one point on AB, it can be inferred that all three are in one plane.

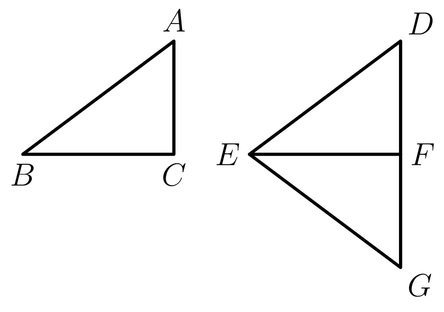
Philo's proof of I. 8.

This alternative proof avoids the use of I. 7, and it is elegant; but it is inconvenient in one respect; since three cases have to be distinguished. Proclus gives the proof in the following order (pp. 266, 15–268, 14).

Let ABC, DEF be two triangles having the sides AB, AC equal to the sides DE, DF respectively, and the base BC equal to the base EF.

Let the triangle ABC be applied to the triangle DEF, so that B is placed on E and BC on EF, but so that A falls on the opposite side of EF from D, taking the position G. Then C will coincide with F, since BC is equal to EF.

Now FG will either be in a straight line with DF, or make an angle with it, and in the latter case the angle will either be *interior* (κατὰ τὸ ἐντός) to the figure or *exterior* (κατὰ τὸ ἐκτός).

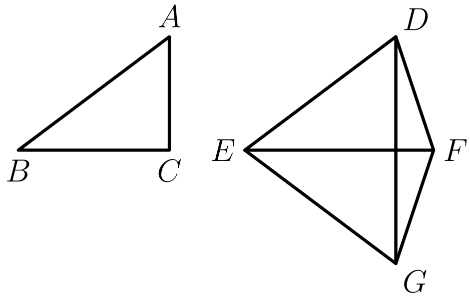


I. Let FG be in a straight line with DF.

Then, since DE is equal to EG, and DFG is a straight line, DEG is an isosceles triangle, and the angle at D is equal to the angle at G. [I. 5].

II. Let DF, FG form an angle *interior* to the figure.

Let DG be joined.



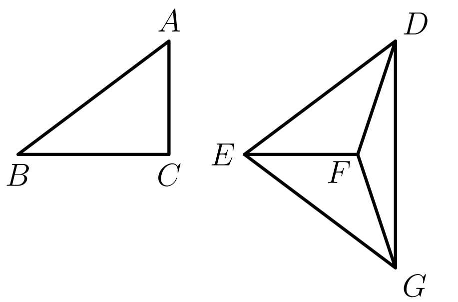
Then, since DE, EG are equal, the angle EDG is equal to the angle EGD.

Again, since DF is equal to FG, the angle FDG is equal to the angle FGD.

Therefore, by addition,
the whole angle EDF is equal to the whole angle EGF.

III. Let DF, FG form an angle *exterior* to the figure.

Let DG be joined.



The proof proceeds as in the last case, except that subtraction takes the place of addition, and the remaining angle EDF is equal to the remaining angle EGF.

Therefore in all three cases the angle EDF is equal to the angle EGF, that is, to the angle BAC.

It will be observed that, in accordance with the practice of the Greek geometers in not recognising as an "angle" any angle not less than two right angles, the re-entrant angle of the quadrilateral $DEGF$ is ignored and the angle DFG is said to be *outside* the figure.