[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 244–246 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 2.]

10. **(as an extremity)**. I have inserted these words because "to place a straight line *at* a given point" (πρὸς τῷ δοθέντι σημείῳ) is not quite clear enough, at least in English.

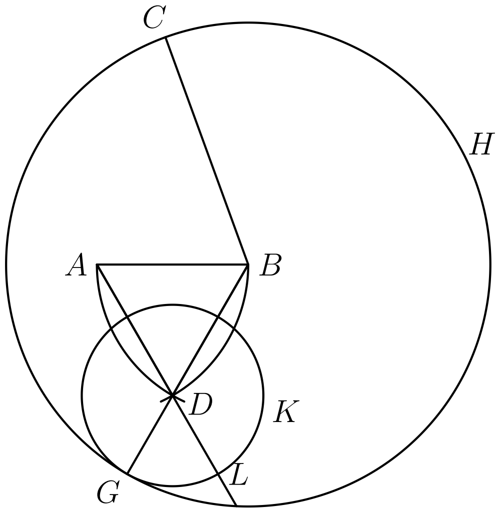
11. **Let the straight lines AE, BF be produced** . . . . It will be observed that in this first application of Postulate 2, and again in I. 5, Euclid speaks of the *continuation* of the straight line as that which is produced in such cases, ἐκβεβλήσθωσαν and προσεκβεβλήσθωσαν meaning little more than *drawing* straight lines "in a straight line with" the given straight lines. The first place in which Euclid uses phraseology exactly corresponding to ours when speaking of a straight line being produced is in I. 16: "let one side of it, BC, be produced to D" (προσεκβεβλήσθω αὐτοῦ μία πλευρὰ ἡ ΒΓ ἐπὶ τὸ Δ).

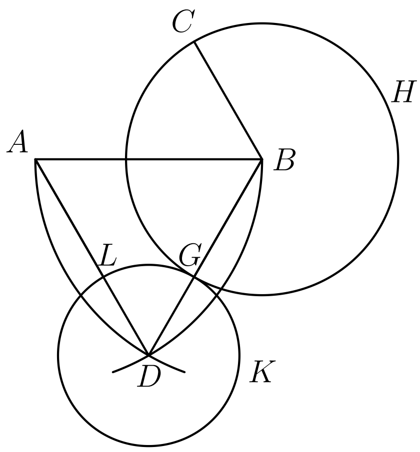
13. **the remainder AL . . . the remainder BG**. The Greek expressions are λοιπὴ ἡ ΑΛ and λοιπῇ τῇ ΒΗ, and the literal translation would be "*AL* (or *BG*) *remaining*," but the shade of meaning conveyed by the position of the definite article can hardly be expressed in English.

This proposition gives Proclus an opportunity, such as the Greek commentators revelled in, of distinguishing a multitude of *cases*. After explaining that those theorems and problems are said to have *cases* which have the same force, though admitting of a number of different figures, and preserve the same method of demonstration while admitting variations of position, and that cases reveal themselves in the *construction*, he proceeds to distinguish the cases in this problem arising from the different positions which the given point may occupy relatively to the given straight line. It may be (he says) either (1) outside the line, or (2) on the line, and if (1), it may be (a) on the line produced or (b) situated obliquely with regard to it; if (2), it may be either (a) one of the extremities of the line or (b) an intermediate point on it. It will be seen that Proclus' anxiety to subdivide leads him to give a "case," (2) (a), which is useless, since in that "case" we are given what we are required to find, and there is really no problem to solve. As Savile says, "qui quaerit ad β punctum ponere rectam aqualem τῇ $\beta\gamma$ rectae, quaerit quad datum est, quod nemo faceret nisi forte insaniat."

Proclus gives the construction for (2) (b) following Euclid's way of taking G as the point in which the circle with centre B intersects DB *produced*, and then proceeds to "cases," of which there are still more, which result from the different ways of drawing the equilateral triangle and of producing its sides.

This last class of "cases" he subdivides into three according as AB is (1) equal to, (2) greater than or (3) less than BC. Here again "case" (1) serves no purpose, since, if AB is equal to BC, the problem is already solved. But Proclus' figures for the other two cases are worth giving, because in one of them the point G is on BD produced beyond D, and in the other it lies on BD itself and there is no need to produce any side of the equilateral triangle. A glance at these figures will show that, if they were used in the proposition,





each of them would require a slight modification in the wording (1) of the construction, since BD is in one case produced beyond D instead of B and in the other case not produced at all, (2) of the proof, since BG, instead of being the difference between DG and DB, is in one case the sum of DG and DB and in the other the difference between DB and DG.

Modern editors generally seem to classify the cases according to the possible variations in the construction rather than according to differences in the data. Thus Lardner, Potts, and Todhunter distinguish eight cases due to the three possible alternatives, (1) that the given point may be joined to either end of the given straight line, (2) that the equilateral triangle may then be described on either side of the joining line, and (3) that the side of the equilateral triangle which is produced may be produced in either direction. (But it should have been observed that, where AB is greater than BC, the third alternative is between producing DB and not producing it at all.) Potts adds that, when the given point lies either on the line or on the line produced, the distinction which arises from joining the two ends of the line with the given point no longer exists, and there are only four cases of the problem (I think he should rather have said *solutions*).

To distinguish a number of cases in this way was foreign to the really classical manner. Thus, as we shall see, Euclid's methodd is to give one case only, for choice the most difficult, leaving the reader to supply the rest for himself. Where there was a real distinction between cases, sufficient to necessitate a substantial difference in the proof, the practice was to give separate *enunciations* and proofs altogether, as we may see, e.g., from the *Conics* and the *De sectione rationalis* of Apollonius.

Proclus alludes, in conclusion, to the error of those who proposed to solve I 2 by describing a circle with the given point as centre and with a distance *equal* to BC, which, as he says, is a *petitio principii.* De Morgan puts the matter very clearly (*Supplementary Remarks on the first six Books of Euclid's Elements* in the *Companion to the Almanac*, 1849, p. 6). We should "insist," he says, "here upon the restrictions imposed by the first three postulates, which do not allow a circle to be drawn with a compass-carried distance; suppose the compasses to close of themselves the moment they cease to touch the paper. These two propositions [I. 2, 3] extend the power of construction to what it would have been if *all* the usual power of the compasses had been assumed; they are mysterious to all who do not see that postulate iii does not ask for *every use of the compasses.*"