[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 242–243 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Proposition 1.]

1. **On a given finite straight line**. The Greek usage differs from ours in that the definite article is employed in such a phrase as this where we have the indefinite. ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης, "on *the* given finite straight line," i.e. the finite straight line which we choose to take.

3. **Let AB be the given finite straight line**. To be strictly literal we should have to translate in the reverse order "let the given finite straight line be the (straight line) AB"; but this order is inconvenient in other cases where there is more than one datum, e.g. in the *setting-out* of I. 2, "let the given point be A, and the given straight line BC," the awkwardness arising from the omission of the verb in the second clause. Hence I have, for clearness' sake, adopted the other order throughout the book.

8. **let the circle BCD be described**. Two things are here to be noted, (1) the elegant and practically universal use of the perfect passive imperative in constructions, γεγράφθω meaning of course "let it *have been* described" or "suppose it described," (2) the impossibility of expressing shortly in a translation the force of the words in their original order. κύκλος γεγράφθω ὁ ΒΓΔ means literally "let a circle have been described, the (circle, namely, which I denote by) BCD." Similarly we have lower down "let straight lines (namely) the (straight lines) CA, CB be joined," ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ. there seems to be no practicable alternative, in English, but to translate as I have done in the text.

13. **from the point C** . . . . Euclid is careful to adhere to the phraseology of Postulate 1 except that he speaks of "joining" (ἐπεζεύχθωσαν) instead of "drawing" (γράφειν). He does not allow himself to use the shortened expression "let the straight line FC be joined" (without mention of the points F, C) until I. 5.

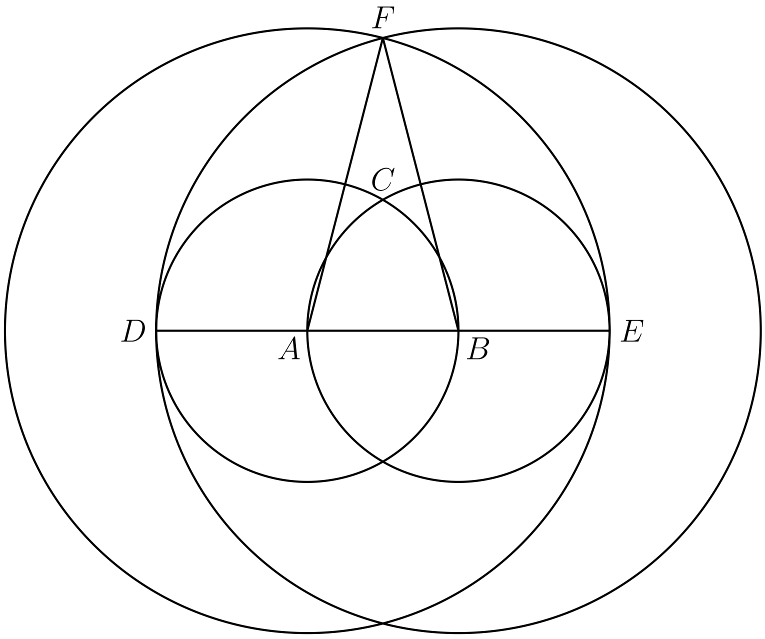
20. **each of the straight lines CA, CB**, ἑκατέρα τῶν ΓΑ, ΓΒ and 24. **the three straight lines CA, AB, BC**, αἱ τρεῖς αἱ ΓΑ, ΑΒ, ΒΓ. I have, here and in all similar expressions, inserted the words "straight lines" which are not in the Greek. The possession of the inflected definite article enables the Greek to omit the words, but this is not possible in English, and it would scarcely be English to write "each of CA, CB" or "the three CA, AB, BC."

It is a commonplace that Euclid has no right to assume, without premising some postulate, that the two circles *will* meet in some point C. To supply what is wanted we must invoke the Principle of Continuity (see notes thereon above, p. 235). It is sufficient for the purpose of this proposition and of I. 22, where there is a similar tacit assumption, to use the form of postulate suggested by Killing. "*if a line* [in this case e.g. the circumference ACE] *belongs entirely to a figure* [in this case a plane] *which is divided into two parts* [namely the part enclosed within the circumference of the circle BCD and the part outside that circle], *and if the line has at least one point common with each part, it must also meet the boundary between the parts* [i.e. the circumference ACE must meet the circumference BCD]."

Zeno's remark that the problem is not solved unless it is taken for granted that two straight lines cannot have a common segment has already been mentioned (note on Post. 2, p. 196). Thus, if AC, BC meet at F before reaching C, and have the part FC common, the triangle obtained, namely FAB, will not be equilateral, but FA, FB will each be less than AB. But Post. 2 has already laid it down that two straight lines cannot have a common segment.

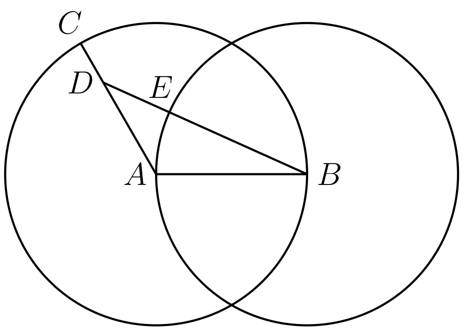
Proclus devotes considerable space to this part of Zeno's criticism, but satisfies himself with the bare mention of the other part, to the effect that it is also necessary to assume that two *circumferences* (with different centres) cannot have a common part. That is, for anything we know, there may be any number of points C common to the two circumferences ACE, BCD. It is not until III. 10 that it is proved that two circles cannot intersect in more points than two, so that we are not entitled to assume it here. The most we can say is that it is enough for the purpose of this proposition if *one* equilateral triangle can be found with the given base; that the construction only gives *two* such triangles has to be left over to be proved subsequently. And indeed we have not long to wait; for I. 7 clearly shows that on either side of the base AB only *one* equilateral triangle can be described. Thus I. 7 gives us the *number of solutions* of which the present problem is susceptible, and it supplies the same want in I. 22 where a triangle has to be described with three sides of given length; that is, I. 7 furnishes us, in both cases, with one of the essential parts of a complete διορισμός, which includes not only the determination of the conditions of possibility but also the number of solutions (ποσαχῶς ἐγχωρεῖ, Proclus, p. 202, 5). This view of I. 7 as supplying an equivalent for III. 10 absolutely needed in I. 1 and I. 22 should serve to correct the idea so common among writers of text-books that I. 7 is merely of use as a lemma to Euclid's proof of I. 8, and therefore may be left out if an alternative proof of that proposition is adopted.

Agreeably to this notion that it is from I. 1 that we must satisfy ourselves that isosceles and scalene triangles actually exist, as well as equilateral triangles, Proclus shows us how to draw, first a particular isosceles triangle, and then a scalene triangle, by means of the figure of the proposition. To make an isosceles triangle he produces AB in both directions to meet the respective circles in D, E, and then describes circles with A, B as centres and AE, BD as radii respectively. The result is an isosceles triangle with each of two sides double of the third side. To make an isosceles triangle in which the equal sides are not so related to the third side but have any given



length would require the use of I. 3; and there is no object in treating the question at all in advance of I. 22. An easier way of satisfying ourselves of the existence of some isosceles triangles would surely be to conceive any two radii of a circle drawn and their extremities joined.

There is more point in Proclus' construction of a *scalene* triangle. Suppose AB to be a radius of one of the two circles, and D a point on AC lying in that portion of the circle with centre A which is outside the circle with centre B. Then, joining BD, as in the figure, we have a triangle which obviously has all its sides unequal, that is, a *scalene* triangle.



The above two constructions appear in an-Nairīzī's commentary under the name of Heron; Proclus does not mention his source.

In addition to the above construction for a scalene triangle (producing a triangle in which the "given" side is greater than one and less than the other of the two remaining sides), Heron has two others showing the other two possible cases, in which the "given" side is (1) less than, (2) greater than, either of the other two sides.