[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 222–223 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Common Notion 1.]

COMMON NOTION 1.

Τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα.

Things which are equal to the same thing are also equal to one another.

Aristotle throughout emphasises the fact that axioms are self-evident truths, which it is impossible to demonstrate. If, he says, any one should attempt to prove them, it could only be through ignorance. Aristotle therefore would undoubtedly have agreed in Proclus' strictures on Apollonius for attempting to prove the axioms. Proclus gives (p. 194, 25), as a specimen of these attempted proofs by Apollonius, that of the first of the *Common Notions.* "Let A be equal to B, and the latter to C; I say that A is also equal to C. For, since A is equal to B, it occupies the same space with it; and since B is equal to C, it occupies the same space with it.

$$A \qquad B \qquad C$$

Therefore A also occupies the same space with C."

Proclus rightly remarks (p. 194, 22) that "the middle term is no more intelligible (better known, $\gamma\nu\omega\rho\mu\omega\sigma\tau\epsilon\rho\sigma\nu$) than the conclusion, if it is not actually more disputable." Again (p. 195, 6), the proof assumes two things, (1) that things which "occupy the same space" ($\tau \delta \pi \sigma \varsigma$) are equal to one another, and (2) that things which occupy the same space with one and the same thing occupy the same space with one another; which is to explain the obvious by something much more obscure, for space is an entity more unknown to us than the things which exist in space.

Aristotle would also have objected to the proof that it is partial and not general ($\varkappa \alpha \vartheta \delta \lambda \circ \nu$), since it refers only to things which can be supposed to occupy a space (or take up room), whereas the axiom is, as Proclus says (p. 196, 1), true of numbers, speeds, and periods of time as well, though of course each science uses axioms in relation to its own subject-matter only.