[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 188–190 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 22.]

Definition 22.

Τῶν δὲ τετραπλεύρων σχημάτων τετράγωνον μέν ἐστιν, ὃ ἰσοπλευρόν τέ ἐστι καὶ ὀρθογώνιον, ἑτερόμηκες δέ, ὃ ὀρθογώνιον μέν, οὐκ ἰσόπλευρον δέ, ῥόμβος δέ, ὃ ἰσόπλευρον μέν, οὐκ ὀρθογώνιον δέ, ῥομβοειδὲς τὸ τὰς ἀπεναντίον πλευράς τε καὶ γωνίας ἴσας ἀλλήλαις ἔχον, ὃ οὔτε ἰσόπλευρόν ἐστιν οὔτε ὀρθογώνιον· τὰ δὲ παρὰ ταῦτα τετράπλευρα τραπέζια καλείσθω.

Of quadrilateral figures, a square *is that which is both equilateral and right-angled, an* oblong *that which is right-angled but not quadrilateral; a* rhombus *that which is equilateral but not right-angled; and a* rhomboid *that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called* trapezia.

τετράγωνον was already a *square* with the Pythagoreans (cf. Aristotle, *Metaph.* 986 a 26), and it is so most commonly in Aristotle; but in *De anima* II. 3, 414 b 31 it seems to be a quadrilateral, and in *Metaph.* 1054 b 2, "equal and equiangular τετράγωνα," is to have any sense. Though, by introducing τετραπλευρον for any quadrilateral, Euclid enabled ambiguity to be avoided, there seem to be traces of the older vague use of τετράγωνον in much later writers. Thus Heron (Def. 100) speaks of a cube as "contained by six equilateral and *equiangular* τετράγωνα" and Proclus (p. 166, 10) adds to his remark about the "four-sided triangle" that 'you might have τετράγωνα with more than the four sides," where τετράγωνα can hardly mean squares.

ἑτερόμηκες, *oblong* (with sides of *different length*), is also a Pythagorean term.

The word *right-angled* (ὀρθογώνιον) as here applied to quadrilaterals must mean *rectangular* (i.e., practically, having all its angles right angles); for, although it is tempting to take the word in the same sense for a square as for a triangle (i.e. "having *one* right angle"), this will not do in the case of the oblong, which, unless it were stated that *three* of its angles are right angles, would not be sufficiently defined.

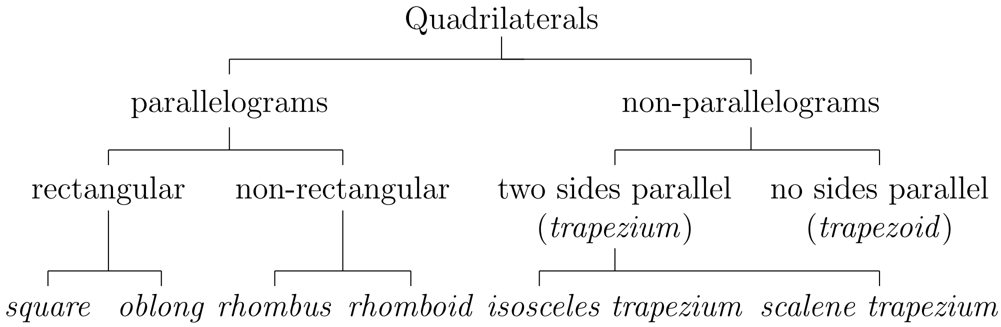
If it be objected, as it was by Todhunter for example, that the definition of a square assumes more than is necessary, since it is sufficient that, being equilateral, it should have one right angle, the answer is that, as in other cases, the superfluity does not matter from Euclid's point of view; on the contrary, the more of the essential attributes of a thing that could be included in its definition the better, provided that the existence of the thing defined and its possession of all those attributes is proved before the definition is actually used; and Euclid does this in the case of the square by construction in I. 46, making no use of the definition before that proposition.

the word *rhombus* (ῥόμβος) is apparently derived from ῥέμβω, to *turn round and round*, and meant among other things a *spinning-top*. Archimedes uses the term *solid rhombus* to denote a solid figure made up of two right cones with a common circular base and vertices turned in opposite directions. We can of course easily imagine this solid generated by *spinning*; and, if the cones were equal, the section through the common axis would be a *plane* rhombus, which would also be the *apparent* form of the spinning solid to the eye. The difficulty in the way of supposing the plane figure to have been named after the solid figure is that in Archimedes the cones forming the solid are not necessarily equal. It is however possible that the solid to which the name was originally given was made up of two equal cones, that the plane rhombus then received its name from that solid, and that Archimedes, in taking up the old name again, extended its signification (cf. J. H. T. Müller, *Beiträge zer Termiologie der griechischen Mathematiker*, 1860, p. 20). Proclus, while he speaks of a rhombus as being like a shaken, i.e. deformed, square, and of a rhomboid as an oblong that has been moved, tries to explain the rhombus by reference to the appearance of a *spinning* square (τετράγωνον ῥομβούμενον).

It is true that the definition of a rhomboid says more than is necessary in describing it as having its opposite sides *and angles* equal to one another. The answer to the objection is the same as the answer to the similar objection to the definition of a square.

Euclid makes no use in the *Elements* of the *oblong*, the *rhombus* and the *rhomboid*. The explanation of his inclusion of definitions of these figures is no doubt that they were taken from earlier text-books. From the words "*let* quadrilaterals other than these *be called* trapezia" we may perhaps infer that *trapezium* was a new name or a new application of an old name.

As Euclid has not yet defined parallel lines and does not anywhere define a *parallelogram*, he is not in a position to make the more elaborate classification of quadrilaterals attributed by Proclus to Posidonius and appearing also in Heron's Definitions. It may be shown by the following diagram, distinguishing seven species of quadrilaterals.



It will be observed that, while Euclid in the above definition classes as *trapezia* all quadrilaterals other than squares, oblongs, rhombi, and rhomboids, the word is in this classification restricted to quadrilaterals having two sides (only) parallel, and *trapezoid* is used to denote the rest. Euclid appears to have used *trapezium* in the restricted sense of a quadrilateral with two sides parallel in his book (περὶ διαιρέσεων (on divisions of figures). Archimedes uses it in the same sense, but in one place describes it more precisely as a trapezium with its two sides parallel.