[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 187–188 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definitions 19, 20, 21.]

Definitions 19, 20, 21.

19. Σχήματα εὐθύγραμμά ἐστι τὰ ὑπὸ εὐθειῶν περιεχόμενα, τρίπλευρα μὲν τὰ ὑπὸ τριῶν, τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων, πολύπλευρα δὲ τὰ ὑπὸ πλειόνων ἢ τεσσάρων εὐθειῶν περιχόμενα.

20. Τῶν δὲ τριπλεύρων σχημάτων ἰσόπλευρον μὲν τρίγωνόν ἐστι τὸ τὰς τρεῖς ἴσας ἔχον πλευράς, ἰσοσκελὲς δὲ τὸ τὰς δύο μόνας ἴσας ἔχον πλευράς, σκαληνὸν δὲ τὸ τὰς τρεῖς ἀνίσους ἔχον πλευράς.

21. Ἔτι δὲ τῶν τριπλεύρων σχημάτων ὀρθογώνιον μὲν τρίγωνόν ἐστι τὸ ἔχον ὀρθὴν γωνίαν, ἀμβλυγώνιον δὲ τὸ ἔχον ἀμβλεῖαν γωνίαν, ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξείας ἔχον γωνίας.

19. Rectilineal figures *are those which are contained by straight lines,* trilateral *figures being those contained by three,* quadrilateral *those contained by four, and* multilateral *those contained by more than four straight lines.*

20. *Of trilateral figures, an* equilateral triangle *is that which has its three sides equal, an* isosceles triangle *that which has two of its sides alone equal, and a* scalene triangle *that which has its three sides unequal.*

21. *Further, of trilateral figures, a* right-angled triangle *is that which has a right angle, and* obtuse-angled triangle *that which has an obtuse angle, and an* acute-angled triangle *that which has its three angles acute.*

19.

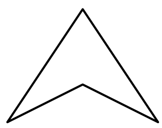
The latter part of this definition, distinguishing *three-sided*, *four-sided* and *many-sided* figures, is probably due to Euclid himself, since the words τρίπλευρον, τετράπλευρον and πολύπλευρον do not apper in Plato or Aristotle (only in one passage of the *Mechanics* and of the *Problems* respectively does even τετράπλευρον, *quadrilateral*, occur). By his use of τετράπλευρον, quadrilateral, Euclid seems practically to have put an end to any ambiguity in the use by mathematicians of the word τετράγωνον, literally "four-angled (figure)," and to have got it restricted to the *square.* Cf. note on Def. 22.

20.

Isosceles (ἰσοσκελής, with equal legs) is used by Plato as well as Aristotle. *Scalene* (σκαληνός, with the variant σκαληνής) is used by Aristotle of a triangle with no two sides equal: cf. also Tim. Locr. 98 B. Plato, *Euthyphro* 12 D, applies the term "scalene" to an *odd* number in contrast to "isosceles" used of an even number. Proclus (p. 168, 24) seems to connect it with σκάζω, to *limp*; others make it akin to σκολιός, *crooked, aslant.* Apollonius uses the same word "scalene" of an *oblique* circular cone.

Triangles are classified, first with reference to their sides, and then with reference to their angles. Proclus points out that seven distinct species of triangles emerge: (1) the *equilateral* triangle, (2) three species of *isosceles* triangles, the right-angled, the obtuse-angled and the acute-angled, (3) the same three varieties of *scalene* triangles.

Proclus gives an odd reason for the dual classification according to sides and angles, namely that Euclid was mindful of the fact that it is not every *triangle* that is *trilateral* also. He explains this statement by reference (p. 165, 22) to a figure which some called *barb-like* (ἀκιδοειδής) while Zenodorus called it *hollow-angled* (κοιλογώνιος). Proclus mentions it again in his note on I. 22 (p. 328, 21 sqq.) as one of the paradoxes of geometry, observing that it is seen in the figure of that proposition. This "triangle" is merely a *quadrilateral*



with a re-entrant angle; and the idea that it has only three angles is due to the non-recognition of the fourth angle (which is greater than two right-angles) as being an angle at all. Since Proclus speaks of the *four-sided triangle* as "one of the paradoxes in geometry," it is perhaps not safe to assume that the misconception underlying the expression existed in the mind of Proclus alone; but there does not seem to be any evidence that Zenodorus called the figure in question a triangle (cf. Pappus, ed. Hultsch, pp. 1154, 1206).