[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 185–186 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 17.]

## Definition 17.

Διάμετρος δὲ τοῦ κύκλου ἐστὶν εὐθεῖά τις διὰ τοῦ κέντρου ἠγμένη καὶ περατουμένη ἐφ᾽ ἑκάτερα τὰ μέρη ὑπὸ τῆς τοῦ κύκλου περιφερείας, ἥτις καὶ δίχα τέμνει τὸν κύκλον.

*A* diameter *of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.*

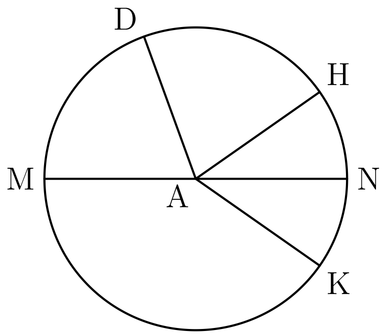
The last words, literally "which (straight line) also bisects the circle," are omitted by Simson and the editors who followed him. But they are necessary even though they do not "belong to the definition" but only express a property of the diameter as defined. For, without this explanation, Euclid would not have been justified in describing as a *semi*-circle a portion of a circle bounded by a diameter and the circumference cut off by it.

Simplicius observes that the *diameter* is so called because it passes *through* the whole surface of a circle as if *measuring* it, and also because it divides the circle into two equal parts. He might however have added that, in general, it is a line passing through a figure where it is *widest*, as well as dividing it equally: thus in Aristotle τὰ κατὰ διάμετρον κείμενα, "things diametrically situated" in space, are at their maximum distance apart. *Diameter* was the regular word in Euclid and elsewhere for the diameter of a *square*, and also of a parallelogram, *diagonal* (γιαγώνιος) was a later term, defined by Heron (Def. 67) as the straight line drawn from an angle to an angle.

Proclus (p. 157, 10) says that Thales was the first to prove that a circle is bisected by its diameter; but we are not told how he proved it. Proclus gives as the *reason* of the property "the undeviating course of the straight line through the centre" (a simple appeal to symmetry), but adds that, if it is desired to prove it mathematically, it is only necessary to imagine the diameter drawn and one part of the circle applied to the other; it is then clear that they must coincide, for, if they did not, and one fell inside or outside the other, the straight lines from the centre to the circumference would not all be equal: which is absurd.

Saccheri's proof is worth quoting. It depend on three "Lemmas" immediately preceding, (1) that two straight lines cannot enclose a space, (2) that two straight lines cannot have one and the same segment common, (3) that, if two straight lines meet at a point, they do not touch, but cut one another, at it.

“Let $MDHNKM$ be a circle, $A$ its centre, $MN$ a diameter. Suppose the portion $MNKM$ of the circle turned about the fixed points $M$, $N$ so that it ultimately comes near to or coincides with the remaining portion $MNHDM$.




“Then (i) the whole diameter $MAN$, with all its points, clearly remains in the same position, since otherwise two straight lines would enclose a space (contrary to the first Lemma).

“(ii) Clearly no point $K$ of the circumference $NKM$ falls within or outside the surface, enclosed by the diameter $MAN$ and the other part, $NHDM$, of the circumference, since otherwise, contrary to the nature of the circle, a radius as $AK$ would be less or greater than another radius as $AH$.

“(iii) Any radius $MA$ can clearly be rectilineally produced only along a single other radius $AN$, since otherwise (contrary to the second Lemma) two lines assumed straight, e.g. $MAN$, $MAH$, would have one and the same common segment.

“(iv) All diameters of the circle obviously cut one another in the centre (Lemma 3 preceding), and they bisect one another there, by the general properties of the circle.

“From all this it is manifest that the diameter $MAN$ divides its circle and the circumference of it just exactly into two equal parts, and the same may be generally asserted for every diameter whatsoever of the same circle; which was to be proved.”

Simson observes that the property is easily deduced from III. 31 and 24; for it follows from III. 31 that the two parts of the circle are “similar segments” of a circle (segments containing equal angles, III. Def. 11), and from III. 24 that they are equal to one another.