[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 183–185 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definitions 15, 26.]

Definitions 15, 16.

15. Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπο μιᾶς γραμμᾶς περιεχόμενον [ἣ καλεῖται περιφέρεια], πρὸς ἣν ἀφ᾽ ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι [πρὸς τὴν τοῦ κύκλου περιφέρειαν] ἴσαι ἀλλήλαις εἰσίν.

16. Κέντρον δὲ τοῦ κύκλου τὸ σημεῖον καλεῖται.

15. *A* circle *is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another*;

16. *And the point is called the* centre *of the circle.*

The words ἣ καλεῖται περιφέρεια, "which is called the circumference," and πρὸς τὴν τοῦ κύκλου περιφέρειαν, "to the circumference of the circle," are bracketed by Heiberg because, although the MSS. have them, they are omitted in other ancient sources, viz. Proclus, Taurus, Sextus Empiricus and Boethius, and Heron also omits the second gloss. The recently discovered papyrus Herculanensis No. 1061 also quotes the definition without the words in question, confirming Heiberg's rejection of them (see Heiberg in *Hermes* XXXVIII., 1903, p. 47). The words were doubtless added in view of the occurrence of the word "circumference" in Deff. 17, 18 immediately following, without any explanation. But no explanation was needed. Though the word περιφέρεια does not occur in Plato, Aristotle uses it several times (1) in the general sense of *contour* without any any special mathematical signification, (2) mathematically, with reference to the rainbow and the circumference, as well as an arc, of a circle. Hence Euclid was perfectly justified in employing the word in Deff. 17, 18 and elsewhere, but leaving it undefined as being a word universally understood and not involving in itself any mathematical conception. It may be added that an-Nairīzī had not the bracketed words in his text; for he comments on and tries to explain Euclid's omission to define the circumference.

The definition itself contains nothing new in substance. Plato (*Parmenides* 137 E) says: "*Round* is, I take it, that the extremes of which are every way equally distant from the middle" (στρογγύλον γέ πού ἐστι τοῦτο, οὗ ἂν τὰ ἔσχατα πανταχῇ ἀπὸ τοῦ μέσου ἴσον ἀπέχῃ). In Aristotle we find the following expressions: "the circular (περιφερόγραμμον) plane figure bounded by one line" (*De caelo* II. 4, 286 b 13–16); "the plane equal (i.e. extending equally all ways) from the middle" (ἐπίπενδον τὸ ἐκ τοῦ μέσου ἴσον), meaning a circle (*Rhetoric* III. 6, 1407 b 27); he also contrasts with the circle "any other figure which has not the lines from the middle equal, as for example an egg-shaped figure" (*De caelo* II. 4, 287 a 19). The word "centre" (κέντρον) was also regularly used: cf. Proclus' quotation from the "oracles" (λόγια), "the centre from which all (lines extending) as far as the rim are equal."

The definition as it stands has no *genetic* character. It says nothing as to the existence or non-existence of the thing defined or as to the method of constructing it. It simply explains what is meant by the word "circle," and is a provisional definition which cannot be used until the existence of circles is proved or assumed. Generally, in such a case, existence is proved by actual construction; but here the possibility of constructing the circle as defined, and consequently its existence, are *postulated* (Postulate 3). A *genetic* definition might state that a circle is the figure described when a straight line, always remaining in one plane, moves about one extremity as a fixed point until it returns to its first position (so Heron, Def. 27).

Simplicius indeed, who points out that the distance between the feet of a pair of compasses is a straight line from the centre to the circumference, will have it that Euclid intended by this definition to show how to construct a circle by the revolution of a straight line about one end as centre; and an-Nairīzī points to this as the explanation (1) of Euclid's definition of a circle as a *plane figure*, meaning the whole surface bounded by the circumference, and not the circumference itself, and (2) of his omission to mention the "circumference," since with this construction the circumference is not drawn separately as a *line*. But it is not necessary to suppose the Euclid himself did more than follow the traditional view; for the same conception of the circle as a *plane figure* appears, as we have seen, in Aristotle. While, however, Euclid is generally careful to say the "*circumference* of a circle" when he means the circumference, or an arc, only, there are cases where "circle" means "circumference of a circle," e.g. in III. 10: "A circle does not cut a circle in more points than two."

Heron, Proclus and Simplicius are all careful to point out that the centre is not the only point which is equidistant from all points of the circumference. The centre is the only point *in the plane of the circle* ("lying within the figure," as Euclid says) of which this is true; any point not in the same plane which is equidistant from all points of the circumference is a *pole*. If you set up a "gnomon" (an upright stick) at the centre of a circle (i.e. a line through the centre perpendicular to the plane of the circle), its upper extremity is a pole (Proclus, p. 153, 3); the perpendicular is the locus of all such poles.