## [Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 182–183 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 14.]

## Definition 14.

Σχῆμά ἐστι τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενον. A figure is that which is contained by any boundary or boundaries.

Plato in the *Meno* observes that roundness ( $\sigma\tau\rho\sigma\gamma\nu\lambda\delta\tau\eta\varsigma$ ) or the round is a "figure," and that the straight and many other things are so too; he then inquires what there is common to all of them, in virtue of which we apply the term "figure" to them. His answer is (76 A): "with reference to every figure I say that that in which the solid terminates (toũto, εἰς ὃ τὸ στερεὸν περαίνει) is a figure, or, to put it briefly, a figure is an extremity of a solid." The first observation is similar to Aristotle's in the *Physics* I. 5, 188 a 25, where *angle*, straight and circular are mentioned as genera of figure. In the Categories 8, 10 a 11, "figure" is placed with straightness and curvedness in the category of quality. Here however "figure" appears to mean shape  $(\mu o \rho \phi \eta)$  rather than "figure" in our sense. Coming nearer to "figure" in our sense, Aristotle admits that figure is "a sort of magnitude" (De anima III. 1, 425 a 18), and he distinguishes *plane figures* of two kinds, in language not unlike Euclid's, as *contained* by straight and circular lines respectively: "every plane figure is either rectilineal or formed by circular lines ( $\pi \epsilon \rho i \varphi \epsilon \rho \phi \gamma \rho \alpha \mu \mu \sigma \nu$ ), and the rectilineal figure is contained by several lines, the circular by one line" (Decaelo II. 4, 286 b 13). He is careful to explain that a plane is not a figure, nor a figure a plane, but that a plane figure constitutes one notion and is a species of the genus figure (Anal. post. II. 3, 90 b 37). Aristotle does not attempt to define figure in general, in fact he says it would be useless: "From this it is clear that there is one definition of soul in the same way as there is one definition of *figure*; for in the one case there is no figure except the triangle, quadrilateral, and so on, nor is there any soul other than those above mentioned. A definition might be constructed which should apply to all figures but not specially to any particular figure, and similarly with the species of soul referred to. But such a general definition would serve no purpose.] Hence it is absurd here as elsewhere to seek a general definition which will not be properly a definition of anything in existence and will not be applicable to the particular irreducible species before us, to the neglect of the definition which is so applicable" ( $De \ anima \ II. \ 3, \ 414 \ b \ 20-28$ ).

Comparing Euclid's definition with the above, we observe that by introducing *boundary* ( $\delta\rho\sigma\varsigma$ ) he at once excludes the *straight* which Aristotle classed as figure; he doubtless excluded *angle* also, as we may judge by (1) Heron's statement that "neither one nor two straight lines can complete a figure," (2) the alternative definition of a straight line as "that which cannot with another line of the same species form a figure," (3) Geminus' distinction between the line which forms a figure ( $\sigma\chi\eta\mu\alpha\tau\sigma\pi\sigma\iota\sigma\sigma\sigma\sigma\sigma\alpha$ ) and the line which extends indefinitely ( $\dot{\epsilon}\pi$ '  $\dot{\alpha}\pi\epsilon\iota\rho\sigma\nu$   $\dot{\epsilon}\kappa\beta\alpha\lambda\lambda\circ\mu\dot{\epsilon}\nu\eta$ ), which latter term includes a hyperbola and a parabola. Instead of calling figure an extremity as Plato did in the expression "extremity (or limit) of a solid," Euclid describes a figure as that which has a boundary or boundaries. And lastly, in spite of Aristotle's objection, he does attempt a general definition to cover all kinds of figure, solid and plane. It appears certain therefore that Euclid's definition is entirely his own.

Another view of a figure, recalling that of Plato in *Meno* 76 A, is attributed by Proclus (p. 143, 8) to Posidonius. The latter regarded the *figure* as the *confining extremity or limit* ( $\pi \epsilon \rho \alpha \zeta \sigma \upsilon \gamma \varkappa \lambda \epsilon \tilde{\iota} \sigma \upsilon \rangle$ ), "separating the notion of figure from *quantity* (or magnitude) and making it the cause of *definition*, *limitation*, and *inclusion* ( $\tau \circ \tilde{\upsilon} \& \rho (\sigma \vartheta \alpha \varkappa \alpha) \pi \epsilon \pi \epsilon \rho (\sigma \vartheta \alpha \varkappa \alpha) \tau \tilde{\eta} \zeta \pi \epsilon \rho \iota \circ \tilde{\eta} \zeta )...$ Posidonius thus seems to have in view only the boundary placed round from outside, Euclid the whole content, so that Euclid will speak of the circle as a figure in respect of its whole plane (surface) and of its inclusion (from) without, whereas Posidonius (makes it a figure) in respect of its circumference... Posidonius wished to explain the notion of figure as itself *limiting* and *confining* magnitude."

Proclus observes that a logical and refining critic might object to Euclid's definition as defining the genus from the species, since that which is enclosed by one boundary and that which is enclosed by several are both species of figure. The best answer to this seems to be supplied by the passage of Aristotle's *De anima* quoted above.