[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), p. 171 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 6.]

Definition 6.

Ἐπιφανείας δὲ πέρατα γραμμαί.

The extremities of a surface are lines.

It being unscientific, as Aristotle says, to define a line as the extremity of a surface, Euclid avoids the error of defining the prior by means of the posterior in this way, and gives a different definition not open to this objection. Then, by way of compromise, and in order to show the connexion between a line and a surface, he adds the equivalent of the definition of a line previously current as an explanation.

As in the corresponding definition Def. 3 above, he omits to add what is made clear by Aristotle (*Metaph.* 1060 b 15) that a "division" (διαίρεσις) or "section" (τομή) of a solid or body is also a surface, or that the common boundary at which two parts of a solid fit together (*Categories* 6, 5 a 2) may be a surface.

Proclus discusses how the fact stated in Def. 6 can be said to be true of surfaces like that of the sphere "which is bounded (πεπέρασται), it is true, but not by lines." His explanation (p. 116, 8–14) is that, "if we take the surface (of a sphere), so far as it is extended two ways (διχῇ διαστατή), we shall find that it is bounded by lines as to length and breadth; and if we consider the spherical surface as possessing a form of its own and invested with a fresh quality, we must regard it as having fitted end on to beginning and made the two ends (or extremities) one, being thus one potentially only, and not in actuality."