[Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), pp. 169–170 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 5.]

Definition 5.

Ἐπιφάνεια δέ ἐστιν, ὃ μῆκος καὶ πλάτος μόνον ἔχει.
A surface *is that which has length and breadth only.*

The word ἐπιφάνεια was used by Euclid and later writers to denote *surface* in general, while they appropriated the word ἐπίπεδον for *plane* surface, thus making ἐπίπεδον a *species* of the *genus* ἐπιφάνεια. A solitary use of ἐπιφάνεια by Euclid when a plane is meant (xi. Def. 11) is probably due to the fact that the particular definition came from an earlier textbook. Proclus (p. 116, 17) remarks that the older philosophers, including Plato and Aristotle, used the words ἐπιφάνεια and ἐπίπεδον indifferently for any kind of surface. Aristotle does indeed use both words for a surface, with perhaps a tendency to use ἐπιφάνεια more than ἐπίπεδον for a surface not plane. Cf. *Categories* 6, 5 a 1 sq., where both words are used in one sentence: "You can find a common boundary at which the parts fit together, a point in the case of a line, and a line in the case of a surface (ἐπιφάνεια); for the parts of the surface (ἐπιπέδου) do fit together at some common boundary. Similarly also in the case of a body you can find a common boundary, a line or a surface (ἐπιφάνεια), at which the parts of the body fit together." Plato however does not use ἐπιφάνεια at all in the sense of surface, but only ἐπίπεδον for both *surface* and *plane surface*. There is reason therefore for doubting the correctness of the notice in Diogenes Laertius, iii. 24, that Plato "was the first philosopher to name, among extremities, the *plane* surface" (ἐπίπεδος ἐπιφάνεια).

ἐπιφάνεια of course means literally the feature of a body which is *apparent* to the eye (ἐπιφανής), namely the surface.

Aristotle tells us (*De sensu* 3, 439 a 31) that the Pythagoreans called a surface χροιά, which seems to have meant *skin* as well as *colour*. Aristotle explains the term with reference to colour (χρῶμα) as a thing inseparable from the extremity (πέρας) of a body.

Alternative definitions.

The definitions of a surface correspond to those of a line. As in Aristotle a line is a magnitude "(extended) one way, or in one 'dimension'" (ἐφ᾽ ἕν), "continuous one way" (ἐφ᾽ ἓν συνεχές), or "divisible in one way" (μοναχῇ διαιρετόν), so a surface is magnitude extended or continuous *two ways* (ἐπὶ δύο), or divisible *in two ways* (διχῇ). As in Euclid a surface has "length and breadth" only, so in Aristotle "breadth" is characteristic of the surface and is once used as synonymous with it (*Metaph.* 1020 a 12), and again "lengths are made up of long and short, *surfaces of broad and narrow*, and solids (ὄγκοι) of deep and shallow" (*Metaph.* 1085 a 10).

Aristotle mentions the common remark that *a line by its motion produces a surface* (*De anima* I. 4, 409 a 4). He also gives the *a posteriori* description of a surface as the "extremity of a solid" (*Topics* VI. 4, 141 b 22), and as "the section (τομή) or division (διαίρεσις) of a body" (*Metaph.* 1060 b 14).

Proclus remarks (p. 144, 20) that we get a notion of a surface when we measure areas and mark their boundaries in the sense of length and breadth; and we further get a sort of perception of it by looking at shadows, since these have no depth (for they do not penetrate the earth) but only have length and breadth.

Classification of surfaces.

Heron gives (Def. 74, p. 50, ed. Heiberg) two alternative divisions of surfaces into two classes, corresponding to Geminus' alternative divisions of lines, viz. into (1) *incomposite* and *composite* and (2) *simple* and *mixed.*

(1) *Incomposite* surfaces are "those which, when produced, fall into (or coalesce with) themselves" (ὅσαι ἐκβαλλόμεναι αὐταὶ καθ᾽ ἑαυτῶν πίπτουσιν), i.e. are of continuous curvature, e.g. the sphere.

Composite surfaces are "those which, when produced, cut one another." Of composite surfaces, again, some are (*a*) made up of non-homogeneous (elements) (ἐξ ἀνομοιογενῶν) such as cones, cylinders and hemispheres, others (*b*) made up of homogeneous (elements), namely the rectilineal (or polyhedral) surfaces.

(2) Under the alternative division, *simple* surfaces are the plane and the spherical surfaces, but no others; the *mixed* class includes all other surfaces whatever and is therefore infinite in variety.

Heron specially mentions as belonging to the mixed class (*a*) the surface of cones, cylinders and the like, which are a mixture of plane and circular (μικταὶ ἐξ ἐπιπέδου καὶ περιφερείας) and (b) *spiric* surfaces, which are "a mixture of two circumferences" (by which he must mean a mixture of two circular elements, namely the generating circle and its circular motion about an axis in the same plane).

Proclus adds the remark that, curiously enough, *mixed* surfaces may arise by the revolution either of *simple* curves, e.g. in the case of the *spire*, or of *mixed* curves, e.g. the "right-angled conoid" from a parabola, "another conoid" from the hyperbola, the "oblong" (ἐπίμηκες, in Archimedes παραμᾶκες) and "flat" (ἐπιπλατύ) spheroids from an ellipse according as it revolves around the major or minor axis respectively (pp. 119, 6–120, 2). The *homoeomeric* surfaces, namely those any part of which will coincide with any other part, are *two* only (the plane and the spherical surface), not three as in the case of lines (p. 120, 7).