## [Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd edition), p. 165 (1925).]

[Heath's commentary on Euclid, *Elements*, Book I, Definition 3.]

## DEFINITION 3.

<sup>°</sup>Γραμμῆς δὲ πέρατα σημεῖα. *The extremities of a line are points.* 

It being unscientific, as Aristotle said, to define a point as the "extremity of a line" ( $\pi \epsilon \rho \alpha \varsigma \gamma \rho \alpha \mu \mu \tilde{\eta} \varsigma$ ), thereby explaining the prior by the posterior, Euclid defined a point differently; then, as it was necessary to connect a point with a line, he introduced this explanation after the definitions of both had been given. This compromise is no doubt his own idea; the same thing occurs with reference to a surface and a line as its extremity in Def. 6, and with reference to a solid and a surface as its extremity in XI. Def. 2.

We miss a statement of the facts, equally requiring to be known, that a "division" (διαίρεσις) of a line, no less than its "beginning" or "end," is a point (this is brought out by Aristotle: cf. Metaph. 1060 b 15), and that the *intersection* of two lines is also a point. If these additional explanations had been given, Proclus would have been spared the difficulty which he finds in the fact that some of the lines used in Euclid (namely infinite straight lines on the one hand, and circles on the other) have no "extremities." So also the ellipse, which Proclus calls by the old name  $\vartheta \upsilon \rho \varepsilon \delta \zeta$  ("shield"). In the case of the circle and ellipse we can, he observes (p. 103, 7), take a portion bounded by points, and the definition applies to that portion. His rather far-fetched distinction between two aspects of a circle or ellipse as a line and as a closed figure (thus, while you are describing a circle, you have two extremities at any moment, but they disappear when it is finished) is an unnecessarily elaborate attempt to establish the literal universality of the "definition," which is really no more than an explanation that, if a line has extremities, those extremities are points.