MAU34804—Fixed Point Theorems and Economic Equilibria
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Appendix E: Some Notational Conventions involving Vectors and Matrices

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E. Notational Conventions involving Vectors and Matrices

E.1. Notation for Inequalities involving Vectors

We establish some notation that will be used throughout this section.

Let m and n be positive integers. Given any $m \times n$ matrix T, we denote by $(T)_{i,j}$ the coefficient in the ith row and jth column of the matrix T for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Also given any n-dimensional vector $\mathbf{v}$, we denote by $(\mathbf{v})_j$ the jth coefficient of the vector j for $j = 1, 2, \ldots, n$.

Definition

A matrix T is said to be *non-negative* if all its coefficients are non-negative real numbers.

Definition

A matrix T is said to be *positive* if all its coefficients are strictly positive real numbers.

Let S and T be $m \times n$ matrices. If $(S)_{i,j} \leq (T)_{i,j}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, then we denote this fact by writing $S \leq T$, or by writing $T \geq S$. If $(S)_{i,j} < (T)_{i,j}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, then we denote this fact by writing $S << T$, or by writing $T >> S$.

Let $\mathbf{u}$ and $\mathbf{u}$ be n-dimensional vectors. If $(\mathbf{u})_j \leq (\mathbf{v})_j$ for $j = 1, 2, \ldots, n$, then we denote this fact by writing $\mathbf{u} \leq \mathbf{v}$, or by writing $\mathbf{v} \geq \mathbf{u}$. If $(\mathbf{u})_j < (\mathbf{v})_j$ for $j = 1, 2, \ldots, n$, then we denote this fact by writing $\mathbf{u} << \mathbf{v}$, or by writing $\mathbf{v} >> \mathbf{u}$.

A matrix T with real coefficients is thus *non-negative* if and only if $T \geq 0$. A matrix T with real coefficients is *positive* if and only if $T >> 0$.