[Robert Simson, *Euclid's Elements of Geometry* (5th edition), pp.v–vii. (1775).]

PREFACE

The Opinions of the Moderns concerning the Author of the Elements of Geometry, which go under Euclid's name, are very different and contrary to one another. Peter Ramus ascribes the Propositions, as well as their Demonstrations, to Theon; others think the Propositions to be Euclid's, but that the Demonstrations are Theon's; and others maintain that all the Propositions and their Demonstrations are Euclid's own. John Buteo and Sir Henry Savile are the Authors of greatest Note who assert this last, and the greater part of Geometers have ever since been of this Opinion, as they thought it the most probable. Sir Henry Savile, after the several Arguments he brings to prove it, makes this Conclusion (Page 13. Prælect.) "That, excepting a very few "Interpolations, Explications, and Additions, Theon altered nothing in Euclid." But, by often considering and comparing together the Definitions and Demonstrations as they are in the Greek Editions we now have, I found that Theon, or whoever was the Editor of the present Greek Text, by adding some things, suppressing others, and mixing his own with Euclid's Demonstrations, had changed more things to the worse than is commonly supposed, and those not of small moment, especially in the Fifth and Eleventh Books of the Elements, which this Editor has greatly vitiated; for instance, by substituting a shorter, but insufficient Demonstration of the 18th Prop. of the 5th Book, in place of the legitimate one which Euclid had given; and by taking out of this Book, besides other things, the good Definition which Eudoxus or Euclid had given of Compound Ratio, and giving an absurd one in place of it in the 5th Definition of the 6th Book, which neither Euclid, Archimedes, Appollonius, nor any Geometer before Theon's time, ever made use of, and of which there is not to be found the least appearance in any of their Writings; and, as this Definition did much embarass Beginners, and is quite useless, it is now thrown out of the Elements, and another, which without doubt Euclid had given, is put in its proper place among the Definitions of the 5th Book, by which the Doctrine of Compound Ratios is rendered plain and easy. Besides, among the Definitions of the 11th Book, there is this, which is the 10th, viz. "Equal and similar solid Figures are those which are contained by similar Planes of the same Number and Magnitude." Now, this Proposition is a Theorem, not a Definition; because the equality of Figures of any kind must be demonstrated, and not assumed; and therefore, though this were a true Proposition, it ought to have been demonstrated. But indeed this Proposition, which makes the 10th Definition of the 11th Book, is not true universally, except in the case in which each of the solid Angles of the Figures is contained by no more than three plane Angles; for, in other Cases, two solid Figures may be contained by similar Planes of the same Number and Magnitude, and yet be unequal to one another; as shall be made evident in the Notes subjoined to these Elements. In like manner, in the Demonstration of the 26th Prop. of the 11th Book, it is taken for granted, that those solid Angles are equal to one another which are contained by plane Angles of the same Number and Magnitude, placed in the same Order; but neither is this universally true, except in the case in which the solid Angles are contained by no more than three plain Angles; nor of this Case is there any Demonstration in the Elements we now have, though it be quite necessary there should be one. Now, upon the 10th Definition of this Book depend the 25th and 28th Propositions of it; and, upon the 25th and 26th depend other eight, viz. the 27th, 31st, 32d, 33d, 34th, 36th, 37th, and 40th of the same Book; and the 12th of the 12th Book depends upon the eighth of the same, and this 8th, and the Corollary of Proposition 17th, and Prop. 18th of the 12th Book, depend on the 9th Definition of the 11th Book, which is not a right Definition; because there may be Solids contained by the same number of similar plane Figures, which are not similar to one another, in the true Sense of Similarity received by all Geometers; and all these Propositions have, for these reasons, been insufficiently demonstrated since Theon's time hitherto. Besides, there are several other things, which have nothing of Euclid's Accuracy, and which plainly shew, that his Elements have been much corrupted by unskilful Geometers; and, though these are not so gross as the others now mentioned, they ought by no means to remain uncorrected.

Upon these Accounts it appeared necessary, and I hope will prove acceptable to all Lovers of accurate Reasoning, and of Mathematical Learning, to remove such Blemishes, and restore the principal Books of the Elements to their original Accuracy, as far as I was was able; especially since these Elements are the Foundation of a Science by which the Investigation and Discovery of useful Truths, at least in Mathematical Learning, is promoted as far as the limited Powers of the Mind allow; and which likewise is of the greatest Use in the Arts both of Peace and War, to many of which Geometry is absolutely necessary. This I have endeavoured to do, by taking away the inaccurate and false Reasonings which unskilful Editors have put into the place of some of the genuine Demonstrations of Euclid, who has very been justly celebrated as the most accurate of Geometers, and by restoring to him those Things which Theon and others have suppressed, and which have these many Ages been buried in Oblivion.

In this fifth Edition, Ptolemy's Proposition concerning a Property of quadrilateral Figures in a Circle is added at the End of the sixth Book.

Also the Note on the 29th Prop. Book 1st, is altered, and made more explicit, and a more general Demonstration is given, instead of that which was in the Note on the 10th Definition of Book 11th; besides the Translation is much amended by the friendly Assistance of a learned Gentleman.

To which are also added, the Elements of Plane and Spherical Trigonometry, which are commonly taught after the Elements of Euclid.

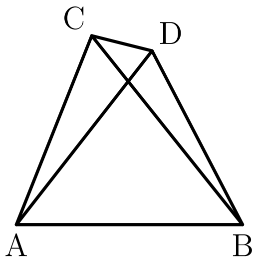
[Robert Simson, *Euclid's Elements of Geometry* (5th edition), pp.19–20 (1775).]

PROP. VII. THEOR.

See N.

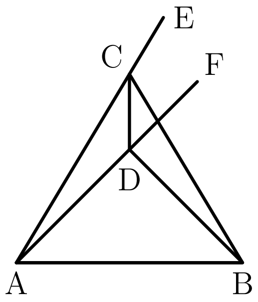
Upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base equal to one another, and likewise those which are terminated in the other extremity.

If it be possible, let there be two triangles ACB, ADB, upon the same base AB, and upon the same side of it, which have their sides CA, DA, terminated in the extremity A of the base, equal to one another, and likewise their sides CB, DB that are terminated in *B*.



Join CD; then, in the case in which the vertex of each of the triangles is without the other triangle, because AC is equal to AD, the angle ACD is equal[a] to the angle ADC: But the angle ACD is greater than the angle BCD; a 5. I.
therefore the angle ADC is greater also than BCD; much more then is the angle BDC greater than the angle BCD. Again, because CB is equal to DB, the angle BDC is equal to the angle BCD; but it has been demonstrated to be greater than it; which is impossible.

But if one of the vertices, as D, be within the other triangle ACB; produce AC, AD to E, F; therefore, because AC is equal to AD in the triangle ACD, the angles ECD, FDC upon the other side of the base CD are equal[b] to one b 5. I.
another; but the angle ECD is greater than the angle BCD; wherefore the angle FCD is likewise greater than BCD; much more then is the angle BDC greater than the angle BCD. Again because CB is equal to DB, the angle BDC is equal[b] to the angle BCD; but BDC has been proved to be greater than the same BCD, which is impossible. The case in which the vertex of one triangle is upon a side of the other needs no demonstration.



Therefore upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base equal to one another, and likewise those which are terminated in the other extremity.

[Extract from Simson's Notes concluding his Edition of the *Elements*]

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.300 (1775).]

PROP. VII. B.I.

There are two cases of this proposition, one of which is not in the Greek text, but is as necessary as the other: And that the case left out has been formerly in the text appears plainly from this, that the second part of prop. 5. which is necessary to the demonstration of this case, can be of no use at all in the elements, or any where else, but in this demonstration; because the second part of prop. 5. clearly follows from the first part, and prop. 13. B.1. This part therefore must have been added to prop. 5. upon account of some proposition betwixt the 5. and 13. but none of these stand in need of it except the 7. proposition, on account of which it has been added: Besides, the translation from the Arabic has this case explicitly demonstrated: And Proclus acknowledges that the second part of prop. 5. was added upon account of prop. 7. but gives a ridiculous reason for it, "that it might afford an answer to objections made against the 7." as if the case of the 7. which is left out, were, as he expressly makes it, an objection against the proposition itself. Whoever is curious may read what Proclus says on this in his commentary on the 5. and 7. propositions; for it is not worth while to relate his trifles at full length.

It was thought proper to change the enunciation of this 7. prop. so as to preserve the very same meaning; the literal translation from the Greek being extremely harsh, and difficult to be understood by beginners.

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.28 (1775).]

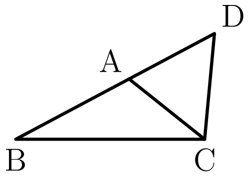
PROP. XX. THEOR.

See N.

Any two sides of a triangle are together greater than the third side.

Let ABC be a triangle; any two sides of it together are greater than the third side, viz. the sides BA, AC greater than the side AC; and AB, BC greater than AC; and BC, CA greater than AB.

Produce BA to the point D, and make[a] AD equal to AC; and join DC. a 3. I.



Because DA is equal to AC, the angle ADC is likewise equal to ACD; but the angle BCD is greater than the angle ACD; therefore the angle BCD is greater than the angle ADC; and because the angle BCD of the triangle DCB is greater than its angle BDC, and that the greater[c] side is opposite to the greater angle; therefore the side DB is greater than the side BC; but DB is equal to BA and AC; therefore the sides BA, AC are greater than BC. In the same manner it may be demonstrated, that the sides AB, BC are greater than CA, and BC, CA greater than AB. Therefore any two sides, &c. Q.E.D. c 19. I.

[**Extract from Simson's Notes concluding his Edition of the *Elements***]

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.301 (1775).]

PROP. XX. and XXI. B.I.

Proclus, in his commentary, relates, that the Epicureans derided this proposition, as being manifest even to asses, and needing no demonstration; and his answer is, that though the truth of it be manifest to our senses, yet it is science which must give the reason why two sides of a triangle are greater than the third: But the right answer to this objection against this and the

21st and some other plain propositions, is, that the number of axioms ought not to be increased without necessity, as it must be if these propositions be not demonstrated. Mons. Clairault, in the preface to his elements of geometry, published in French at Paris, ann. 1741, says, That Euclid has been at the pains to prove, that the two sides of a triangle which is included within another are together less than the two sides of the triangle which includes it; but he has forgot to add this condition, viz. that the triangles must be upon the same base; because, unless this be added, the sides of the included triangle may be greater than the sides of the triangle which includes it, in any ratio which is less than that of two to one, as Pappus Alexandrinus has demonstrated in Prop. 3. b. 3. of his mathematical collections.

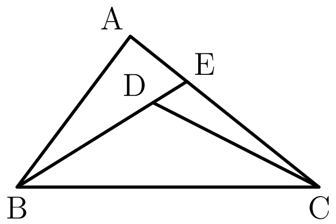
[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.28–29 (1775).]

PROP. XXI. THEOR.

See N.

If, from the ends of the side of a triangle, there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle, but shall contain a greater angle.

Let the two straight lines BD, CD be drawn from B, C, the ends of the side BC of the triangle ABC, to the point D within it; BD and DC are less than the other two sides BA, AC of the triangle, but contain an angle BDC greater than the angle BAC.



Produce BD to E; and because two sides of a triangle are greater than the third side, the two sides BA, AE of the triangle ABE are greater than BE. To each of these add EC; therefore the sides BA, AC are greater than BE, EC: Again, because the two sides CE, ED of the triangle CED are greater than CD, and DB to each of these; therefore the sides CE, EB are greater than CD, DB; but it has been shewn that BA, AC are greater than BE, EC; much more then are BA, AC greater than BD, DC.

Again, because the exterior angle of a triangle is greater than the interior and opposite angle, the exterior angle BDC of the triangle CDE is greater than CED; for the same reason, the exterior angle CEB of the triangle ABE is greater than BAC; and it has been demonstrated that the angle BDC is greater than the angle CEB; much more then is the angle BDC greater than the angle BAC. Therefore, if from the ends of &c. Q.E.D.

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), pp.29–30 (1775).]

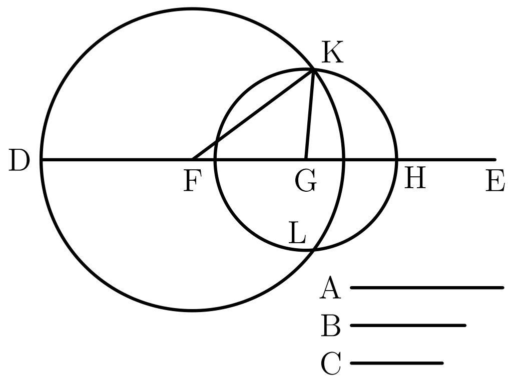
PROP. XXII. PROB.

See N.

To make a triangle of which the sides shall be equal to three given straight lines; but any two whatever of these must be greater than the third[a]. a 20. I.

Let A, B, C be the three given straight lines, of which any two whatever are greater than the third viz. A and B greater than C; A and C greater than B; and B and C than A. It is required to make a triangle of which the sides shall be equal to A, B, C, each to each.

Take a straight line DE terminated at the point D, but unlimited towards E, and make[a] DF equal to A, FG to B, and GH equal to C; and from the centre F, at the distance FD, describe the circle DKL; and from the centre G, at the distance GH, describe another circle HLK, and join KF, KG; the triangle KFG has its sides equal to the three straight lines A, B, C. a 3. I.



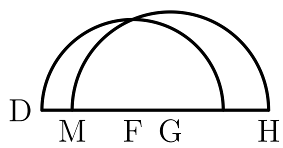
Because the point F is the centre of the circle DKL, FD is equal[c] to FK; but FD is equal to the straight line A; therefore FK is equal to A: Again, because G is the centre of the circle LKH, GH is equal[c] to GK; but GH is equal to C; therefore also GK is equal to C; and FG is equal to B; therefore the three straight lines KF, FG, GK are equal to the three A, B, C: And therefore the triangle KFG has its three sides KF, FG, GK equal to the three given straight lines A, B, C. Which was to be done. c 15. Def.

[**Extract from Simson's Notes concluding his Edition of the *Elements***]

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), pp.301–302 (1775).]

PROP. XXII. B.I.

Some authors blame Euclid because he does not demonstrate, that the two circles made use of in the construction of this problem must cut one another: But this is very plain from the determination he has given, viz. that any two of the straight lines DF, FG, GH must be greater than the third: For who is so dull, tho' only beginning to learn the elements, as not



to perceive that the circle described from the centre F, at the distance FD, must meet FH betwixt F and H, because FD is less than FH; and that, for the like reason, the circle described from the centre G, at the distance GH or GM, must meet DG betwixt D and G; and that these circle must meet one another, because FD and GH are together greater than FG? And this determination is easier to be understood than that which Mr Thomas Simpson derives from it, and puts instead of Euclid's, in the 49th page of his elements of geometry, that he may supply the omission he blames Euclid for; which determination is, that any of the three straight lines must be less than the sum, but greater than the difference of the other two: From this he shews the circles must meet one another, in one case: But the straight line GM which he bids take from GF may be greater than it, as in the figure here annexed; in which case his demonstration must be changed into another.

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), pp.30–31 (1775).]

PROP. XXIV. THEOR.

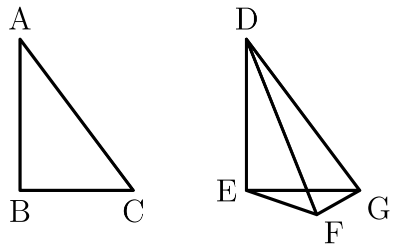
See N.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them, of the other; the base of that which has the greater angle shall be greater than the base of the other.

Let ABC, DEF be two triangles which have the two sides AB, AC equal to the two DE, DF, each to each, viz. AB equal to DE, and AC to DF; but the angle BAC greater than the angle EDF; the base BC is also greater than the base EF.

Of the two sides DE, DF, let DE be the side which is not greater than the other, and at the point D, in the straight line DE, make[a] the angle EDG equal to the angle BAC; and make DG equal[b] to AC or DF, and join EG, GF.

a 23. I.
b 3. I.



Because AB is equal to DE, and AC to DG, the two sides BA, AC are equal to the two ED, DG, each to each, and the angle BAC is equal to the angle EDG; therefore the base BC is equal[c] to the base EG; and because DG is equal to DF, the angle DFG is equal[d] to the angle DGF; but the angle DGF is greater than the angle EGF; therefore the angle DFG is greater than EGF; and much more is the angle EFG greater than the angle EGF; and because the angle EFG of the triangle EFG is greater than the angle EGF, and that the greater[e] side is opposite to the greater angle; the side EG is therefore greater than the side EF; but EG is equal to BC; and therefore also BC is greater than EF. Therefore, if two triangles, &c. Q.E.D.

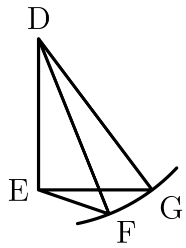
c 4. I.
d 5. I.
e 19. I.

[**Extract from Simson's Notes concluding his Edition of the *Elements***]

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.302 (1775).]

PROP. XXIV. B.I.

To this is added, "of the two sides DE, DF, let DE be that which is not greater than the other;" that is, take that side of the two DE, DF which is not greater than the other, in order to make with it the angle EDG equal to BAC; because, without this restriction, there might be three different cases of the proposition, as Campanus and others make.



Mr Thomas Simpson, in p. 262 of the second edition of his elements geometry, printed ann. 1760, observes, in his notes, that it ought to have been shewn, that the point F falls below the line EG; this probably Euclid omitted, as it is very easy to perceive, that DG being equal to DF, the point G is in the circumference of a circle described from the centre D at the distance DF, and must be in that part of it which is above the straight line EF, because DG falls about DF, the angle EDG being greater than the angle EDF.

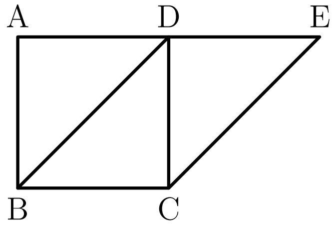
[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.40 (1775).]

PROP. XXXV. THEOR.

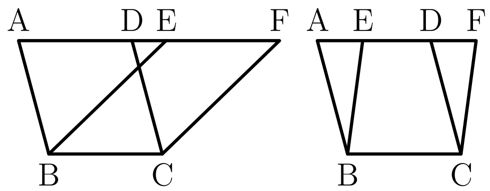
See N.

Parallelograms upon the same base and between the same parallels, are equal to one another.

Let the parallelograms ABCD, EBCF, be upon the same base BC, and between the same parallels AF, BC; the parallelogram ABCD shall be equal to the parallelogram EBCF. See the 2d and 3d figures.



If the sides AD, DF of the parallelograms ABCD, DBCF opposite to the base BC be terminated in the same point D; it is plain that each of the parallelograms is double[a] of the triangle BDC; and they are therefore equal to one another.



But, if the sides AD, EF, opposite to the base BC of the parallelograms ABCD, EBCF, be not terminated in the same point; then, because ABCD is a parallelogram, AD is equal[a] to BC; for the same reason EF is equal to BC; wherefore AD is equal[b] to EF, and DE is common; therefore the whole, or the remainder, *AE*, is equal[c] to the whole, or the remainder DF; AB also is equal to DC; and the two EA, AB are therefore equal to the two FD, DC, each to each; and the exterior angle FDC is equal[d] to the interior EAB; therefore the base EB is equal to the base FC, and the triangle EAB equal[e] to the triangle FDC; take the triangle FDC from the trapezium ABCF, and from the same trapezium take the triangle EAB; the remainders therefore are equal[f] that is, the parallelogram ABCD is equal to the parallelogram EBCF. Therefore parallelographs upon the same base &c. Q.E.D.

b 1. Ax.
c. 2. or 3. Ax.
d 29. I.
e 4. I.
f 3. Ax.

[**Extract from Simson's Notes concluding his Edition of the *Elements***]

[Robert Simson, *Euclid's Elements of Geometry* (5th edition), p.307 (1775).]

PROP. XXXV. B.I.

The demonstration of this Proposition is changed, because, if the method which is used in it was followed, there would be three cases to be separately demonstrated, as is done in the translation from the Arabic; for, in the Elements, no case of a Proposition that requires a different demonstration, ought to be omitted. On this account, we have chosen the method which Mons. Clairault has given, the first of any, as far as I know, in his Elements, page 21. and which afterwards Mr Simpson gives in his page 32. But whereas Mr Simpson makes use of Prop. 26 B. I. from which the equality of the two triangles does not immediately follow, because, to prove that, the 4. of B. I. must likewise be made use of, as may be seen, in the very same case in the 34. Prop. B. I. it was thought better to make use only of the 4. of B. I.